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An Introduction to Supersymmetry

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Abstract

A fairly elementary introduction to supersymmetric field theories in general and the minimal supersymmetric Standard Model (MSSM) in particular is given. Topics covered include the cancellation of quadratic divergencies, the construction of the supersymmetric Lagrangian using superfields, the field content of the MSSM, electroweak symmetry breaking in the MSSM, mixing between different superparticles (current eigenstates) to produce mass eigenstates, and the embedding of the MSSM in so–called minimal supergravity.

1. Introduction

In the last 20 years the SLAC Spires data base has registered almost 10,000 papers dealing with various aspects of supersymmetric field theories. This is quite remarkable, given that there is *no* direct experimental evidence for the existence of any of the numerous new particles predicted by such theories. This apparent discrepancy between theoretical speculation and experimental fact has even caught the public eye, and led to charges that modern particle physics resembles medieval alchemy.

I will therefore start these lecture notes by reviewing in some detail the main argument for the existence of supersymmetric particles "at the weak scale" (i.e., with mass very roughly comparable to those of the heaviest known elementary particles, the W and Z bosons and the top quark). This argument rests on the observation that supersymmetric field theories "naturally" allow to chose the weak scale to be many orders of magnitude below the hypothetical scale M_X of Grand Unification or the Planck scale M_{Pl}. This is closely related to the cancellation of quadratic divergencies [1] in supsersymmetric field theories; such divergencies are notorious in non–supersymmetric theories with elementary scalar particles, such as the Standard Model (SM). In Sec. 2 this question will be discussed in more detail, and the cancellation of quadratic divergencies involving Yukawa interactions will be demonstrated explicitly (in 1–loop order).

This explicit calculation will indicate the basic features that the proposed new symmetry has to have if it is to solve the "naturalness problem" [2] of the SM. In particular, we will need equal numbers of physical (propagating) bosonic and fermionic degrees of freedom; also, certain relations between the coefficients of various terms in the Lagrangian will have to hold. In Sec. 3 we will discuss a method that allows to quite easily construct field theories that satisfy these conditions, using the language of superfields. This will be the most formal part of these notes. At the end of this section, the Lagrangian will have been constructed, and we will be ready to check the cancellation of quadratic divergencies due to gauge interactions. This involves a far greater number of diagrams and fields than the case of Yukawa interactions; it seems quite unlikely that one could have hit on the necessary set of fields and their interactions using the kind of guesswork that will be used (with hindsight) in Sec. 2. At the end of Sec. 3 the problem of supersymmetry breaking will be discussed briefly.

Having hopefully convinced the reader that supersymmetric field theories are interesting, and having shown how to construct them in general, in Sec. 4 I attempt to make contact with reality by discussing several issues related to the phenomenology of the simplest potentially realistic supersymmetric field theory, the Minimal Supersymmetric Standard Model or MSSM. I will begin this section with a review of the motivation for considering a supersymmetrization of the SM. The absence of quadratic divergencies remains the main argument, but the MSSM also has several other nice features not shared by the SM. In Sec. 4a the field content of the model will be listed, and the Lagrangian will be written down; this is an obvious application of the results of Sec. 3. In Sec. 4b the breaking of the electroweak gauge symmetry will be discussed. This plays a central role both theoretically (since without elementary scalar "Higgs" bosons the main argument for weak–scale supersymmetry collapses) and phenomenologically (since it will lead to a firm and, at least in principle, easily testable prediction). Next, mixing between various superparticles ("sparticles") will be discussed. This mixing, which is a direct consequence of $SU(2) \times U(1)_Y$ gauge symmetry breaking, unfortunately makes the correspondence between particles and sparticles less transparent. However, an understandig of sparticle mixing is essential for an understanding of almost all ongoing work on the phenomenology of the MSSM.

The least understood aspect of the MSSM concerns the breaking of supersymmetry. A general parametrization of this (necessary) phenomenon introduces more than 100 free parameters in the model. Fortunately not all of these parameters will be relevant for a given problem or process, at least not in leading order in perturbation theory. Nevertheless, it is of interest to look for schemes that attempt to reduce the number of free parameters. The most popular such scheme is (loosely) based on the extension of global supersymmetry to its local version, supergravity, and is hence known as "minimal supergravity" or mSUGRA. This model is attractive not only because of its economy and resulting predictive power, but also because it leads to a dynamical explanation (as opposed to a mere parametrization) of electroweak symmetry breaking. This will be discussed in Sec. 4d. I will in conclude Sec. 5 by briefly mentioning some areas of active research.

2. Quadratic Divergencies

This section deals with the problem of quadratic divergencies in the SM, and an explicit calculation is performed to illustrate how the introduction of new fields with judicioulsy chosen couplings can solve this problem. In order to appreciate the "bad" quantum behaviour of the scalar sector of the SM, let us first briefly review some corrections in QED, the best understood ingredient of the SM.

The examples studied will all be two–point functions (inverse propagators) at vanishing external momentum, computed at one–loop level. The calculations will therefore be quite simple, yet they suffice to illustrate the problem. Roughly speaking, the computed quantity corresponds to the mass parameters appearing in the Lagrangian; since I will assume vanishing external momentum, this will not be the on–shell (pole) mass, but it is easy to see that the difference between these two quantities can at most involve logarithmic divergencies (due to wave function renormalization).

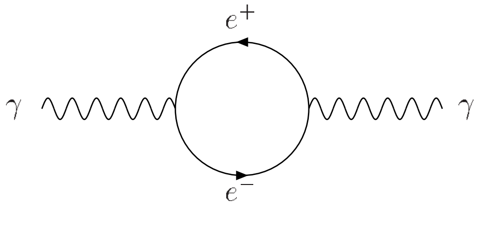

Fig. 1: The photon self–energy diagram in QED.

Let us first investigate the photon's two–point function, which receives contributions due to the electron loop diagram of Fig. 1:

$$\begin{aligned}\pi^{\mu\nu}_{\gamma\gamma}(0) &= -\int \frac{d^4k}{(2\pi)^4}\mathrm{tr}\left[(-ie\gamma^\mu)\frac{i}{\not k - m_e}(-ie\gamma^\nu)\frac{i}{\not k - m_e}\right] \\ &= -4e^2\int \frac{d^4k}{(2\pi)^4}\frac{2k^\mu k^\nu - g^{\mu\nu}\left(k^2-m_e^2\right)}{\left(k^2-m_e^2\right)^2} \\ &= 0. \end{aligned} \tag{1}$$

The fact that the integral in eq.(1) vanishes is manifest only in a regularization scheme that preserves gauge invariance, e.g. dimensional regularization. On a deeper level, this result is the consequence of the exact $U(1)$ gauge invariance of QED, which ensures that the photon remains massless in all orders in perturbation theory.

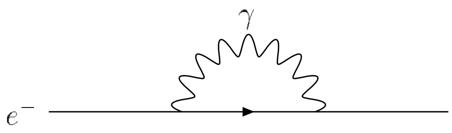

Fig. 2: The electron self energy in QED.

Next, let us consider the electron self energy correction of Fig. 2:

$$\begin{aligned}\pi_{ee}(0) &= \int \frac{d^4k}{(2\pi)^4} \left(-ie\gamma_\mu\right) \frac{i}{\not{k} - m_e} \left(-ie\gamma_\nu\right) \frac{-ig^{\mu\nu}}{k^2} \\ &= -e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2\left(k^2 - m_e^2\right)} \gamma_\mu \left(\not{k} + m_e\right) \gamma^\mu \\ &= -4e^2 m_e \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2\left(k^2 - m_e^2\right)}. \end{aligned} \tag{2}$$

In the last step I have made use of the fact that the $\not{k}$−term in the numerator vanishes after integration over angles, if one uses a regulator that respects Poincaré invariance. The integral in eq.(2) has a logarithmic divergence in the ultraviolet (at large momenta). Notice, however, that this correction to the electron mass is itself proportional to the electron mass. The coefficient is formally infinite; however, even if we replace the "infinity" by the largest scale in particle physics, the Planck scale, we find a correction

$$\delta m_e \simeq 2\frac{\alpha_{\rm em}}{\pi} m_e \log \frac{M_{Pl}}{m_e} \simeq 0.24 m_e, \tag{3}$$

which is quite modest. At a deeper level, the fact that this correction is quite benign can again be understood from a symmetry: In the limit $m_e \to 0$, the model becomes invariant under chiral rotations $\psi_e \to \exp(i\gamma_5\varphi)\psi_e$. If this symmetry were exact, the correction of eq.(2) would have to vanish. In reality the symmetry is broken by the electron mass, so the correction must itself be proportional to m_e.

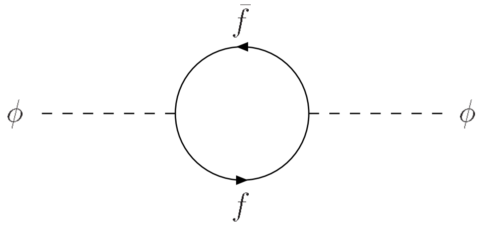

Fig. 3: A fermion anti–fermion contribution to the self energy of the Higgs boson in the Standard Model.

Now consider the contribution of heavy fermion loops to the two–point function of the SM Higgs field $\phi = \Re(H - v)/\sqrt{2}$, shown in Fig. 3. Let the $Hf\bar{f}$ coupling be λ_f; the correction is then given by

$$\begin{aligned}
\pi^f_{\phi\phi}(0) &= -N(f) \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[\left(i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k} - m_f}\left(i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k} - m_f}\right] \\
&= -2N(f)\lambda_f^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2 + m_f^2}{\left(k^2 - m_f^2\right)^2} \\
&= -2N(f)\lambda_f^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{\left(k^2 - m_f^2\right)^2}\right]. \qquad (4)
\end{aligned}$$

Here, $N(f)$ is a multiplicity factor (e.g., $N(t) = 3$ for top quarks, due to summation over color indices).

The first term in the last line of eq.(4) is *quadratically* divergent! If we were to replace the divergence Λ^2 by M_{Pl}^2, the resulting "correction" would be some 30 orders of magnitude *larger* than the physical SM Higgs mass, $m_\phi \leq 1$ TeV (to preserve unitarity of WW scattering amplitudes [3]). The contrast to the modest size of the correction (3) dramatically illustrates the difference between logarithmic and quadratic divergencies. Note also that the correction (4) is itself independent of m_ϕ. This is related to the fact that setting $m_\phi = 0$ does *not* increase the symmetry group of the SM. There is nothing in the SM that "protects" the Higgs mass in the way that the photon and even the electron masses are protected.

Of course, one can still simply renormalize the quadratic divergence away, as one does with logarithmic divergencies. However, I just argued that logarithmic and quadratic divergencies are indeed quite different. Besides, this would still leave us with a finite correction from eq.(4), of order $N(f)m_f^2\lambda_f^2/8\pi$. Such a correction would be quite small if f is an SM fermion like the top quark. However, it is quite unlikely that the SM is indeed the "ultimate theory"; it is much more plausible that at some very high energy scale it will have to be replaced by some more fundamental theory, for example a Grand Unified model [4]. In this case there will be corrections like those of eq.(4), with m_f being of order of this new (very high) scale. One would then need extreme finetuning to cancel a very large bare mass against very large loop corrections, leaving a result of order 1 TeV or less. Moreover, the finetuning would be very different in different orders of perturbation theory. This is the (technical aspect of) the "hierarchy problem": Scalar masses "like" to be close to the highest mass scale in the theory [5].

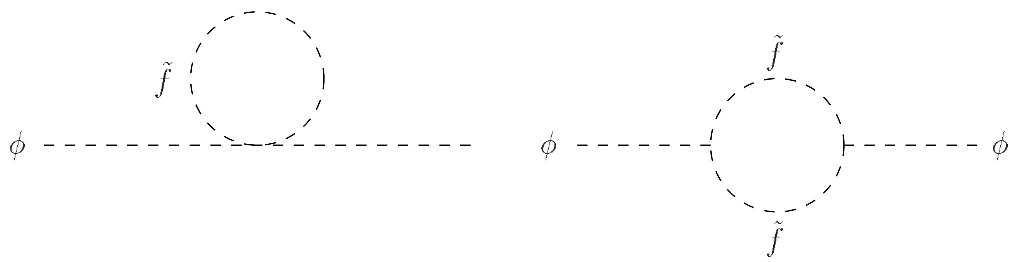

Fig. 4: Sfermion loop contributions to the Higgs self energy. $\tilde{f}$ stands for either $\tilde{f}_L$ or $\tilde{f}_R$.

In supersymmetric field theories this problem is solved since there are additional contributions to $\pi_{\phi\phi}$. For example, let us introduce two complex scalar fields $\tilde{f}_L$, $\tilde{f}_R$, with the following coupling to the Higgs field:

$$\mathcal{L}_{\phi\tilde{f}} = \frac{1}{2}\tilde{\lambda}_f \phi^2 \left(\left|\tilde{f}_L\right|^2 + \left|\tilde{f}_R\right|^2 \right) + v\tilde{\lambda}_f \phi \left(\left|\tilde{f}_L\right|^2 + \left|\tilde{f}_R\right|^2 \right) + \left(\frac{\lambda_f}{\sqrt{2}} A_f \phi \tilde{f}_L \tilde{f}_R^{\,*} + h.c. \right). \tag{5}$$

Here, v is the vacuum expectation value of the SM Higgs field ($v \simeq 246$ GeV). The second term in the Lagrangian (5) is thus due to the breaking of $SU(2) \times U(1)_Y$, and its coefficient is related to that of the first term. The coefficient of the third term is left arbitrary, however. (The factor λ_f appears here just by convention.) This Lagrangian gives the following contribution to $\pi_{\phi\phi}$ (I assume the multiplicity factor N to be the same for $\tilde{f}_L$ and $\tilde{f}_R$):

$$\begin{aligned}
\pi^{\tilde{f}}_{\phi\phi}(0) = & -\tilde{\lambda}_f N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2 - m^2_{\tilde{f}_L}} + \frac{1}{k^2 - m^2_{\tilde{f}_R}} \right] \\
& + \left(\tilde{\lambda}_f v\right)^2 N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{\left(k^2 - m^2_{\tilde{f}_L}\right)^2} + \frac{1}{\left(k^2 - m^2_{\tilde{f}_R}\right)^2} \right] \\
& + \left|\lambda_f A_f\right|^2 N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k^2 - m^2_{\tilde{f}_L}\right)\left(k^2 - m^2_{\tilde{f}_R}\right)}
\end{aligned} \tag{6}$$

Only the first line in eq.(6), which comes from the left diagram in Fig. 4, contains quadratically divergent terms. Comparing this with the fermionic contribution of eq.(4), we see that the quadratic divergencies can be made to cancel by choosing

$$N(\tilde{f}_L) = N(\tilde{f}_R) = N(f); \tag{7a}$$

$$\tilde{\lambda}_f = -\lambda_f^2. \tag{7b}$$

(Note that $\tilde{\lambda}_f < 0$ is required for the scalar potential to be bounded from below.)

Notice that the cancellation of quadratic divergencies does not impose any restrictions on the masses $m_{\tilde{f}_L}$, $m_{\tilde{f}_R}$, nor on the coupling A_f. Let us now sum eqs.(4) and (6) after eqs.(7) have been imposed. To this end, I will use $\overline{MS}$ (or $\overline{DR}$ [6]) regularization for all divergencies, which gives:

$$\int \frac{d^4k}{i\pi^2} \frac{1}{k^2 - m^2} = m^2 \left(1 - \log \frac{m^2}{\mu^2} \right); \tag{8a}$$

$$\int \frac{d^4k}{i\pi^2} \frac{1}{\left(k^2 - m^2\right)^2} = -\log \frac{m^2}{\mu^2}, \tag{8b}$$

where μ is the renormalization scale. For simplicity, let us assume $m_{\tilde{f}_L} = m_{\tilde{f}_R} \equiv m_{\tilde{f}}$; the result can easily be generalized. This gives:

$$\begin{aligned}
\pi^{f+\tilde{f}}_{\phi\phi}(0) = i\frac{\lambda_f^2 N(f)}{16\pi^2} \Bigg[& -2m_f^2 \left(1 - \log \frac{m_f^2}{\mu^2} \right) + 4m_f^2 \log \frac{m_f^2}{\mu^2} \\
& + 2m^2_{\tilde{f}} \left(1 - \log \frac{m^2_{\tilde{f}}}{\mu^2} \right) - 4m_f^2 \log \frac{m^2_{\tilde{f}}}{\mu^2}
\end{aligned}$$

$$-|A_f|^2 \log \frac{m_{\tilde{f}}^2}{\mu^2} \Bigg] . \tag{9}$$

Here, the first line is the fermionic contribution of eq.(4), and the next three terms corresponds to the three lines of eq.(6); in the next–to–last term, I have used the relation $m_f = \lambda_f v/\sqrt{2}$, which is true for SM fermions. From eq.(9) we see that we can achieve a complete cancellation between the fermionic and bosonic contributions, i.e. a *vanishing* total correction, if we require in addition to eqs.(8):

$$m_{\tilde{f}} = m_f; \tag{10a}$$

$$A_f = 0. \tag{10b}$$

The fact that this leads to a vanishing total correction strongly hints at the existence of an additional symmetry, as the discussion of QED radiative corrections at the beginning of this section shows. This line of reasoning will be pursued in the next section. Before we turn to this, let us see what happens if we violate the conditions (10) only a little, i.e. if we take $m_{\tilde{f}}^2 = m_f^2 + \delta^2$, with $\delta,\ |A_f| \ll m_f$, so that $\log \frac{m_{\tilde{f}}^2}{\mu^2} \simeq \log \frac{m_f^2}{\mu^2} + \frac{\delta^2}{m_f^2}$:

$$\begin{aligned} \pi_{\phi\phi}^{f+\tilde{f}} &\simeq i \frac{\lambda_f^2 N(f)}{16\pi^2} \left[-2\delta^2 \log \frac{m_f^2}{\mu^2} - 4\delta^2 - |A_f|^2 \log \frac{m_f^2}{\mu^2} \right] + \mathcal{O}(\delta^4, A_f^2 \delta^2) \\ &= -i \frac{\lambda_f^2 N(f)}{16\pi^2} \left[4\delta^2 + \left(2\delta^2 + |A_f|^2 \right) \log \frac{m_f^2}{\mu^2} \right] + \mathcal{O}(\delta^4, A_f^2 \delta^2). \end{aligned} \tag{11}$$

(The first term in the first line of eq.(10) comes from the first and third terms in eq.(9), and the second term from the second and fourth terms.) We thus find the remarkable result that even if we send m_f to infinity, the correction (11) will remain of modest size as long as the *difference* between m_f^2 and $m_{\tilde{f}}^2$, as well as the coefficient $|A_f|$, remain small. Thus the introduction of the fields $\tilde{f}_L$ and $\tilde{f}_R$ has not only allowed us to cancel quadratic divergencies; it also shields the weak scale from loop corrections involving very heavy particles, *provided* the mass splitting between fermions and bosons is itself of the weak scale.[1]

3. Construction of Supersymmetric Field Theories

This section describes how to construct the Lagrangian of a supersymmetric field theory. To that end I first give a formal definition of the supersymmetry (SUSY) algebra. The next two subsections introduce chiral and vector superfields, respectively. The construction of a Lagrangian with exact

[1] The alert reader will have noticed that I cheated a little. In the derivation of eq.(9) I used $m_f = \lambda_f v/\sqrt{2}$. Sending $m_f \to \infty$ then amounts to to sending $\lambda_f \to \infty$, if v is fixed to its SM value; in this case the correction (11) is still large, and perturbation theory becomes altogether unreliable. However, the main result survives in a more careful treatment of models with two very different mass scales: The low scale is shielded from the high scale as long as the mass splitting between bosons and fermions is itself only of the order of the low scale. This already follows from dimensional considerations, once we have shown that the corrections vanish entirely in the limit where eqs.(10) hold exactly.

supersymmetry will be accomplished in Sec. 3d. In the following subsection it will be shown that this Lagrangian also leads to the cancellation of quadratic divergencies from one–loop gauge contributions to the Higgs two–point function $\pi_{\phi\phi}(0)$, thereby extending the result of the previous section. Finally, soft SUSY breaking is treated in Sec. 3f.

Many excellent reviews of and introductions to the material covered here already exist [6, 7]; I will therefore be quite brief. My notation will mostly follow that of Nilles [7].

3a. The SUSY Algebra

We saw in Sec. 2 how contributions to the Higgs two–point function $\pi_{\phi\phi}(0)$ coming from the known SM fermions can be cancelled exactly, if we introduce new bosonic fields with judiciously chosen couplings. This strongly indicates that a new symmetry is at work here, which can protect the Higgs mass from large (quadratically divergent) radiative corrections, something that the SM is unable to do.[2] We are thus looking for a symmetry that can *enforce* eqs.(7) and (10) (as well as their generalizations to gauge interactions). In particular, we need equal numbers of physical (propagating) bosonic and fermionic degrees of freedom, eq.(7a). In addition, we need relations between various terms in the Lagrangian involving different combinations of bosonic and fermionic fields, eqs.(7b) and (10).

It is quite clear from these considerations that the symmetry we are looking for must connect bosons and fermions. In other words, the generators Q of this symmetry must turn a bosonic state into a fermionic one, and vice versa. This in turn implies that the generators themselves carry half–integer spin, i.e. are fermionic. This is to be contrasted with the generators of the Lorentz group, or with gauge group generators, all of which are bosonic. In order to emphasize the new quality of this new symmetry, which mixes bosons and fermions, it is called *supersymmetry* (SUSY).

The simplest choice of SUSY generators is a 2–component (Weyl) spinor Q and its conjugate $\overline{Q}$. Since these generators are fermionic, their algebra can most easily be written in terms of anti–commutators:

$$\{Q_\alpha, Q_\beta\} = \left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} = 0; \tag{12a}$$

$$\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu; \qquad [Q_\alpha, P_\mu] = 0. \tag{12b}$$

Here the indices α, β of Q and $\dot\alpha$, $\dot\beta$ of $\overline{Q}$ take values 1 or 2, $\sigma^\mu = (\mathbf{1}, \sigma_i)$ with σ_i being the Pauli matrices, and P_μ is the translation generator (momentum); it must appear in eq.(12b) for the SUSY algebra to be consistent with Lorentz covariance [10].

For a compact description of SUSY transformations, it will prove convenient to introduce "fermionic coordinates" $\theta, \overline{\theta}$. These are anti–commuting, "Grassmann" variables:

$$\{\theta, \theta\} = \left\{\theta, \overline{\theta}\right\} = \left\{\overline{\theta}, \overline{\theta}\right\} = 0. \tag{13}$$

A "finite" SUSY transformation can then be written as $\exp\left[i(\theta Q + \overline{Q\theta} - x_\mu P^\mu)\right]$; this is to be compared with a non–abelian gauge transformation $\exp\left(i\varphi_a T^a\right)$, with T^a being the group generators.

[2] In principle one can cancel the one–loop quadratic divergencies in the SM without introducing new fields, by explicitly cancelling bosonic and fermionic contributions; this leads to a relation between the Higgs and top masses [8]. However, such a cancellation would be purely "accidental", not enforced by a symmetry. It is therefore not surprising that this kind of cancellation *cannot* be achieved once corrections from two or more loops are included [9].

Of course, the objects on which these SUSY transformations act must then also depend on θ and $\overline{\theta}$. This leads to the introduction of *superfields*, which can be understood to be functions of θ and $\overline{\theta}$ as well as the spacetime coordinates x_μ. Since θ and $\overline{\theta}$ are also two–component spinors, one can even argue that supersymmetry doubles the dimension of spacetime, the new dimensions being fermionic.

For most purposes it is sufficient to consider infinitesimal SUSY transformations. These can be written as

$$\delta_S(\alpha,\overline{\alpha})\Phi(x,\theta,\overline{\theta}) = \left[\alpha\frac{\partial}{\partial\theta} + \overline{\alpha}\frac{\partial}{\partial\overline{\theta}} - i\left(\alpha\sigma_\mu\overline{\theta} - \theta\sigma_\mu\overline{\alpha}\right)\frac{\partial}{\partial x_\mu}\right]\Phi(x,\theta,\overline{\theta}), \tag{14}$$

where Φ is a superfield and α, $\overline{\alpha}$ are again Grassmann variables. This corresponds to the following explicit representation of the SUSY generators:

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu; \quad \overline{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} + i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu. \tag{15}$$

It will prove convenient to introduce SUSY–covariant derivatives, which anti–commute with the SUSY transformation (14):

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu; \quad \overline{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} - i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu. \tag{16}$$

Note that eqs.(14)–(16) imply that α and θ have mass dimension $-1/2$, while Q and D have mass dimension $+1/2$.

Eqs.(14)–(16) have been written in a form that treats θ and $\overline{\theta}$ on equal footing. It is often more convenient to use "chiral" representations, where θ and $\overline{\theta}$ are treated slightly differently (I am suppressing spinor indices from now on):

$$\begin{gathered}
\text{L} - \text{representation}: \\
\delta_S\Phi_L = \left(\alpha\frac{\partial}{\partial\theta} + \overline{\alpha}\frac{\partial}{\partial\overline{\theta}} + 2i\theta\sigma^\mu\overline{\alpha}\partial_\mu\right)\Phi_L; \\
D_L = \frac{\partial}{\partial\theta} + 2i\sigma^\mu\overline{\theta}\partial_\mu; \quad \overline{D}_L = -\frac{\partial}{\partial\overline{\theta}}.
\end{gathered} \tag{17a}$$

$$\begin{gathered}
\text{R} - \text{representation}: \\
\delta_S\Phi_R = \left(\alpha\frac{\partial}{\partial\theta} + \overline{\alpha}\frac{\partial}{\partial\overline{\theta}} - 2i\alpha\sigma^\mu\overline{\theta}\partial_\mu\right)\Phi_R; \\
\overline{D}_R = -\frac{\partial}{\partial\overline{\theta}} - 2i\theta\sigma^\mu\partial_\mu; \quad D_R = \frac{\partial}{\partial\theta}.
\end{gathered} \tag{17b}$$

Clearly, $\overline{D}$ (D) has a particularly simple form in the L (R) representation. The following identity allows to switch between representations:

$$\Phi(x,\theta,\overline{\theta}) = \Phi_L(x_\mu + i\theta\sigma_\mu\overline{\theta},\theta,\overline{\theta}) = \Phi_R(x_\mu - i\theta\sigma_\mu\overline{\theta},\theta,\overline{\theta}). \tag{18}$$

So far everything has been written for arbitrary superfields Φ. However, we will only need two kinds of special superfields, which are *irreducible* representations of the SUSY algebra; they will be discussed in the following two subsections.

3b. Chiral Superfields

The first kind of superfield we will need are *chiral superfields*. This name is derived from the fact that the SM fermions are chiral, that is, their left– and right–handed components transfer differently under $SU(2) \times U(1)_Y$. We therefore need superfields with only two physical fermionic degrees of freedom, which can then describe the left– or right–handed component of an SM fermion. Of course, the same superfields will also contain bosonic partners, the *sfermions*.

The simplest way to construct such superfields is to require either

$$\overline{D}\Phi_L \equiv 0 \qquad (\Phi_L \text{ is left} - \text{chiral}) \qquad \text{or} \tag{19a}$$

$$D\Phi_R \equiv 0 \qquad (\Phi_R \text{ is right} - \text{chiral}). \tag{19b}$$

Clearly these conditions are most easily implemented using the chiral representations of the SUSY generators and SUSY–covariant derivatives. For example, eq.(17a) shows that in the L–representation, eq.(19a) simply implies that Φ_L is independent of $\overline{\theta}$, i.e. Φ_L only depends on x and θ. Recalling that θ is an anti–commuting Grassmann variable, eq.(13), we can then expand Φ_L as:

$$\Phi_L(x, \theta) = \phi(x) + \sqrt{2}\theta^\alpha \psi_\alpha(x) + \theta^\alpha \theta^\beta \epsilon_{\alpha\beta} F(x), \tag{20}$$

where summation over identical upper and lower indices is understood, and $\epsilon_{\alpha\beta}$ is the anti–symmetric tensor in two dimensions. Recall that θ has mass dimension $-1/2$. Assigning the usual mass dimension $+1$ to the scalar field ϕ then gives the usual mass dimension $+3/2$ for the fermionic field ψ, and the unusual mass dimension $+2$ for the scalar field F; the superfield Φ itself has mass dimension $+1$. The expansion (20) is exact, since θ only has two components, and eq.(13) implies that the square of any one component vanishes; hence there cannot be any terms with three or more factors of θ. The fields ϕ and F are complex scalars, while ψ is a Weyl spinor. At first glance, Φ_L seems to contain four bosonic degrees of freedom and only two fermionic ones; however, we will see later on that not all bosonic fields represent physical (propagating) degrees of freedom. The expression for Φ_R in the R–representation is very similar; one merely has to replace θ by $\overline{\theta}$.[3]

Applying the explicit form (17a) of the SUSY transformation to the left–chiral superfield (20) gives:

$$\begin{aligned}\delta_S \Phi_L &= \sqrt{2}\alpha^\alpha \psi_\alpha + 2\alpha^\alpha \theta^\beta \epsilon_{\alpha\beta} F + 2i\theta^\alpha \sigma^\mu_{\alpha\dot{\beta}} \overline{\alpha}^{\dot{\beta}} \partial_\mu \phi + 2\sqrt{2} i \theta^\alpha \sigma^\mu_{\alpha\dot{\beta}} \overline{\alpha}^{\dot{\beta}} \theta^\beta \partial_\mu \psi_\beta \\ &\equiv \delta_S \phi + \sqrt{2}\theta \delta_S \psi + \theta\theta \delta_S F. \end{aligned} \tag{21}$$

The first two terms of the first line of eq.(21) come from the application of the $\partial/\partial\theta$ part of δ_S, while the last two terms come from the ∂_μ part; note that the ∂_μ part applied to the last term in eq.(20) vanishes, since it contains three factors of θ. The second line of eq.(21) is just the statement that the SUSY algebra should close, i.e. a SUSY transformation applied to a left–chiral superfield should again give a left–chiral superfield. It is easy to see that this is true, since the first line of eq.(21) does not contain any terms $\propto \overline{\theta}$, so an expansion as in eq.(20) must be applicable to it.

[3] Note that one can also write a left–chiral superfield using the right–chiral representation of the SUSY generators, and vice versa. The physical content of the fields remains the same, of course, but the expressions become quite a bit more lengthy. Nevertheless we will be forced to do this on one later occasion.

Explicitly, we have:

$$\delta_S \phi = \sqrt{2}\alpha\psi \qquad \text{(boson} \to \text{fermion)} \tag{22a}$$

$$\delta_S \psi = \sqrt{2}\alpha F + i\sqrt{2}\sigma^\mu \overline{\alpha}\partial_\mu \phi \qquad \text{(fermion} \to \text{boson)} \tag{22b}$$

$$\delta_S F = -i\sqrt{2}\partial_\mu \psi \sigma^\mu \overline{\alpha} \qquad (F \to \text{total derivative}) \tag{22c}$$

Notice in particular the result (22c); it implies that $\int d^4x F(x)$ is *invariant* under SUSY transformations, assuming as usual that boundary terms vanish. We will come back to this point in Sec. 3d.

3c. Vector Superfields

The chiral superfields introduced in the previous subsection can describe spin–0 bosons and spin–1/2 fermions, e.g. the Higgs boson and the quarks and leptons of the SM. However, we also have to describe the spin–1 gauge bosons of the SM. To this end one introduces *vector superfields* V. They are constrained to be self–conjugate:

$$V(x,\theta,\overline{\theta}) \equiv V^\dagger(x,\theta,\overline{\theta}). \tag{23}$$

This leads to the following representation of V in component form:

$$\begin{aligned} V(x,\theta,\overline{\theta}) = & \left(1 + \frac{1}{4}\theta\theta\overline{\theta\theta}\partial_\mu\partial^\mu\right) C(x) + \left(i\theta + \frac{1}{2}\theta\theta\sigma^\mu\overline{\theta}\partial_\mu\right)\chi(x) + \frac{i}{2}\theta\theta\left[M(x) + iN(x)\right] \\ & + \left(-i\overline{\theta} + \frac{1}{2}\overline{\theta\theta}\sigma^\mu\theta\partial_\mu\right)\overline{\chi}(x) - \frac{i}{2}\overline{\theta\theta}\left[M(x) - iN(x)\right] \\ & - \theta\sigma_\mu\overline{\theta}A^\mu(x) + i\theta\theta\overline{\theta\lambda}(x) - i\overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x). \end{aligned} \tag{24}$$

Here, C, M, N and D are real scalars, χ and λ are Weyl spinors, and A^μ is a vector field. If A^μ is to describe a gauge boson, V must transform as an adjoint representation of the gauge group.

The general form (24) is rather unwieldy. Fortunately, we now have many more gauge degrees of freedom than in nonsupersymmetric theories, since now the gauge parameters are themselves superfields. A general non–abelian supersymmetric gauge transformation acting on V can be described by

$$e^{gV} \longrightarrow e^{-ig\Lambda^\dagger} e^{gV} e^{ig\Lambda} \tag{25}$$

where $\Lambda(x,\theta,\overline{\theta})$ is a chiral superfield and g is the gauge coupling. In the case of an abelian gauge symmetry, this transformation rule can be written more simply as

$$V \longrightarrow V + i(\Lambda - \Lambda^\dagger) \qquad \text{(abelian case).} \tag{26}$$

Remembering that a chiral superfield contains four scalar (bosonic) degrees of freedom as well as one Weyl spinor, it is quite easy to see that one can use the transformation (25) or (26) to chose

$$\chi(x) = C(x) = M(x) = N(x) \equiv 0. \tag{27}$$

This is called the "Wess–Zumino" (W–Z) gauge; it is in some sense the SUSY analog of the unitary gauge in "ordinary" field theory, since it removes many unphysical degrees of freedom. Notice, however, that we have only used three of the four bosonic degrees of freedom in Λ. We therefore still have the "ordinary" gauge freedom, e.g. according to $A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \varphi(x)$ for an abelian theory. In other words, the W–Z gauge can be used in combination with any of the usual gauges. However, the choice (27) is sufficient by itself to remove the first two lines of eq.(24), leading to a much more compact expression for V. Assigning the usual mass dimension $+1$ to A^μ gives the canonical mass dimension $+3/2$ for the fermionic field λ, while the field D has the unusual mass dimension $+2$, just like the $F-$component of the chiral superfield (20). Notice also that the superfield V itself has no mass dimension.

Applying a SUSY transformation to eq.(24) obviously gives a lengthier expression than in case of chiral superfields. Here I only quote the important result

$$\delta_S D = -\alpha \sigma^\mu \partial_\mu \overline{\lambda} + \overline{\alpha} \sigma^\mu \partial_\mu \lambda, \tag{28}$$

which shows that the D component of a vector superfield transforms into a total derivative. Together with the analogous result (22c) for chiral superfields, this provides the crucial clue for the construction of the Lagrangian, to which we turn next.

3d. Construction of the Lagrangian

We are now ready to attempt the construction of the Lagrangian of a supersymmetric field theory. By definition, we want the action to be invariant under SUSY transformations:

$$\delta_S \int d^4 x \mathcal{L}(x) = 0. \tag{29}$$

This is satisfied if $\mathcal{L}$ itself transforms into a total derivative. We saw in eqs.(22a) and (28) that the highest components (those with the largest number of θ and $\overline{\theta}$ factors) of chiral and vector superfields satisfy this requirement; they can therefore be used to construct the Lagrangian. We can thus write the action S schematically as

$$S = \int d^4 x \left(\int d^2\theta \mathcal{L}_F + \int d^2\theta d^2\overline{\theta} \mathcal{L}_D \right), \tag{30}$$

where the integration over Grassmann variables is defined as:

$$\int d\theta_\alpha = 0, \qquad \int \theta_\alpha d\theta_\alpha = 1 \tag{31}$$

(no summation over α). $\mathcal{L}_F$ and $\mathcal{L}_D$ in eq.(30) are general chiral and vector superfields, giving rise to "F–terms" and "D–terms", respectively.

In order to make this more explicit, let us compute the product of two left–chiral superfields:

$$\begin{aligned} \Phi_{1,L} \Phi_{2,L} &= \left(\phi_1 + \sqrt{2}\theta\psi_1 + \theta\theta F_1 \right) \left(\phi_2 + \sqrt{2}\theta\psi_2 + \theta\theta F_2 \right) \\ &= \phi_1\phi_2 + \sqrt{2}\theta \left(\psi_1\phi_2 + \phi_1\psi_2 \right) + \theta\theta \left(\phi_1 F_2 + \phi_2 F_1 - \psi_1\psi_2 \right). \end{aligned} \tag{32}$$

(Recall that $\theta\theta\theta = 0$.) Notice that this is itself a left–chiral superfield, since it does not depend on $\overline{\theta}$, so it is a candidate for a contribution to the $\mathcal{L}_F$ term in the action (30). Indeed, the very last term in eq.(32) looks like a fermion mass term! We have thus identified a first possible contribution to the Lagrangian.

Of course, if the product of two left–chiral superfields is a left–chiral superfield, by induction the same must be true for the product of any number of left–chiral superfields. Let us therefore compute the highest component in the product of three such fields:

$$\int d^2\theta \Phi_{1,L}\Phi_{2,L}\Phi_{3,L} = \phi_1\phi_2 F_3 + \phi_1 F_2 \phi_3 + \phi_1\phi_2 F_3 - \psi_1\phi_2\psi_3 - \phi_1\psi_2\psi_3 - \psi_1\psi_2\phi_3. \tag{33}$$

Note that the last three terms in eq.(33) describe Yukawa interactions between one scalar and two fermions; in the SM such interactions give rise to quark and lepton masses. We have thus identified our first interaction term in the SUSY Lagrangian! Notice that if we, e.g., call ϕ_1 the Higgs field, and ψ_2 and ψ_3 the left– and right–handed components of the top quark, respectively[4], eq.(33) will not only produce the desired Higgs–top–top interaction, but also interactions between a scalar top $\tilde{t}$, the fermionic "higgsino" $\tilde{h}$, and the top quark, with *equal* strength. This is a first example of relations between couplings enforced by supersymmetry.

So far we have identified terms that can give rise to explicit fermion masses, eq.(30), as well as Yukawa interactions, eq.(32), but we have not yet found any terms with derivatives, i.e. kinetic energy terms. Clearly multiplying even more left–chiral superfields with each other is not going to help; it is easy to see that this gives rise to terms with mass dimension > 4 in the Lagrangian, which lead to non–renormalizable interactions. Let us instead consider the product of a left–chiral superfield and its conjugate. The latter is in fact a right–chiral superfield. Since we have to use the same representation of the SUSY generators everywhere, we first have to write this right–chiral superfield in the L–representation, using eq.(18):

$$\begin{aligned}[\Phi_L(x,\theta)]^\dagger &= \phi^* - 2i\theta\sigma_\mu\overline{\theta}\partial^\mu\phi^* - 2\left(\theta\sigma_\mu\overline{\theta}\right)\left(\theta\sigma_\nu\overline{\theta}\right)\partial^\mu\partial^\nu\phi^* \\ &+ \sqrt{2\theta\psi} - 2\sqrt{2}i\left(\theta\sigma_\mu\overline{\theta}\right)\partial^\mu\left(\overline{\theta\psi}\right) + \overline{\theta\theta}F^*. \end{aligned} \tag{34}$$

Clearly the product $\Phi_L\Phi_L^\dagger$ is self–conjugate, i.e. it is a vector superfield. It is therefore a candidate contribution to the "D–terms" in the action (30):

$$\int d^2\theta d^2\overline{\theta}\Phi_L\Phi_L^\dagger = FF^* - \phi\partial_\mu\partial^\mu\phi^* - i\overline{\psi}\sigma_\mu\partial^\mu\psi. \tag{35}$$

This contains kinetic energy terms for the scalar component ϕ as well as the fermionic component ψ of chiral superfields! Equally importantly, eq.(35) does *not* contain kinetic energy terms for F. This field does therefore not propagate; it is a mere auxiliary field, which can be integrated out exactly using its purely algebraic equation of motion. A chiral superfield therefore only has two *physical* bosonic degrees of freedom, described by the complex scalar ϕ, i.e. it contains equal numbers of propagating bosonic and fermionic degrees of freedom.

In order to illustrate how the $F-$fields can be removed from the Lagrangian, let us introduce the *superpotential* f:

$$f(\Phi_i) = \sum_i k_i\Phi_i + \frac{1}{2}\sum_{i,j} m_{ij}\Phi_i\Phi_j + \frac{1}{3}\sum_{i,j,k} g_{ijk}\Phi_i\Phi_j\Phi_k, \tag{36}$$

where the Φ_i are all left–chiral superfields, and the k_i, m_{ij} and g_{ijk} are constants with mass dimension 2, 1 and 0, respectively. The contributions to the Lagrangian that we have identified so far can

[4] More exactly, ψ_3 describes the left–handed anti–top.

be written compactly as

$$\begin{aligned}
\mathcal{L} &= \sum_i \int d^2\theta d^2\overline{\theta} \Phi_i \Phi_i^\dagger + \left[\int d^2\theta f(\Phi_i) + h.c. \right] \\
&= \sum_i \left(F_i F_i^* + |\partial_\mu \phi|^2 - i\overline{\psi}_i \sigma_\mu \partial^\mu \psi_i \right) \\
&+ \left[\sum_j \frac{\partial f(\phi_i)}{\partial \phi_j} F_j - \frac{1}{2} \sum_{j,k} \frac{\partial^2 f(\phi_i)}{\partial \phi_j \partial \phi_k} \psi_j \psi_k + h.c. \right]. \qquad (37)
\end{aligned}$$

Note that in the last line of eq.(37), f is understood to be a function of the scalar fields ϕ_i, rather than of the superfields Φ_i. Using eq.(36) it is easy to convince oneself that the last line in eq.(37) indeed reproduces the previous results (32) and (33). Let us now integrate out the auxiliary fields F_j. Their equations of motion are simply given by $\partial \mathcal{L} / \partial F_j = 0$, which implies

$$F_j = - \left[\frac{\partial f(\phi_i)}{\partial \phi_j} \right]^* . \qquad (38)$$

Plugging this back into eq.(37) gives:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \left[\sum_{j,k} \frac{\partial^2 f(\phi_i)}{\partial \phi_j \partial \phi_k} \psi_j \psi_k + h.c. \right] - \sum_j \left| \frac{\partial f(\phi_i)}{\partial \phi_j} \right|^2 , \qquad (39)$$

where $\mathcal{L}_{\text{kin}}$ stands for the second line in eq.(37). The second term in the Lagrangian (39) describes fermion masses and Yukawa interactions, while the last term describes scalar mass terms and scalar interactions. Since both terms are determined by the single function f, there are clearly many relations between coupling constants.

Before elaborating on this last point, we introduce gauge interactions. The coupling of the gauge (super)fields to the (chiral) matter (super)fields is accomplished by a SUSY version of the familiar "minimal coupling":

$$\begin{aligned}
&\int d^2\theta d^2\overline{\theta} \Phi^\dagger \Phi \longrightarrow \int d^2\theta d^2\overline{\theta} \Phi^\dagger e^{2gV} \Phi \\
&\qquad = |D_\mu \phi|^2 - i\overline{\psi} \sigma_\mu D^\mu \psi + g\phi^* D\phi + ig\sqrt{2} \left(\phi^* \lambda \psi - \overline{\lambda \psi} \phi \right) + |F|^2. \qquad (40)
\end{aligned}$$

In the second step I have used the W–Z gauge (27), and introduced the usual gauge–covariant derivative $D_\mu = \partial_\mu + igA_\mu^a T_a$, where the T_a are group generators. Note that this piece of the Lagrangian not only describes the interactions of the matter fields (both fermions and scalars) with the gauge fields, but also contains gauge–strength Yukawa–interactions between fermions (or higgsinos) ψ, sfermions (or Higgs bosons) ϕ, and gauginos λ.

Finally, the kinetic energy terms of the gauge fields can be described with the help of the superfield

$$W_\alpha = \left(\overline{D}_{\dot\alpha} \overline{D}_{\dot\beta} \epsilon^{\dot\alpha \dot\beta} \right) e^{-gV} D_\alpha e^{gV}; \qquad (41)$$

the $D, \overline{D}$ appearing here are again SUSY–covariant derivatives, which carry spinor subscripts. For abelian symmetries, this reduces to $W_\alpha = \left(\overline{D}_{\dot\alpha} \overline{D}_{\dot\beta} \epsilon^{\dot\alpha \dot\beta} \right) D_\alpha V$. Since $\overline{D}_{\dot\alpha} \overline{D}_{\dot\alpha} \equiv 0$, $\overline{D}_{\dot\alpha} W_\alpha = 0$, so W_α is a left–chiral superfield; its behaviour under gauge transformations is identical to that of e^{gV}, see eq.(25). One can show that the product $W_\alpha W^\alpha$ is gauge invariant; as shown earlier, it is also a left–chiral superfield, so its $\theta\theta$ component may appear in the Lagrangian:

$$\frac{1}{32g^2}W_\alpha W^\alpha = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \frac{1}{2}D_a D^a + \left(-\frac{i}{2}\lambda^a\sigma_\mu\partial^\mu\overline{\lambda}_a + \frac{1}{2}g f^{abc}\lambda_a\sigma_\mu A^\mu_b\overline{\lambda}_c + h.c.\right). \tag{42}$$

In addition to the familiar kinetic energy term for the gauge fields, this also contains a kinetic energy terms for the gauginos λ_a, as well as the canonical coupling of the gauginos to the gauge fields, which is determined by the group structure constants f^{abc}.

Note that eq.(42) does not contain a kinetic energy term for the D_a fields. They are therefore also auxiliary fields, and can again easily be integrated out. From eqs.(40) and (42) we see that their equation of motion is

$$D_a = -g\sum_{i,j}\phi_i^* T_a^{ij}\phi_j, \tag{43}$$

where the group indices have been written explicitly. (The field D in eq.(40) is equal to $\sum_a D_a T^a$, in complete analogy to the gauge fields.) The third term in the second line of eq.(40) and the second term in eq.(42) then combine to give a contribution

$$-V_D = -\frac{1}{2}\sum_a\left|\sum_{i,j} g\phi_i^* T^a_{ij}\phi_j\right|^2 \tag{44}$$

to the scalar interactions in the Lagrangian; these interactions are completely fixed by the gauge couplings. This completes the construction of the Lagrangian for a renormalizable supersymmetric field theory.

3e. Quadratic Divergencies, Part 2

As a first application of the results of the previous subsection, let us check that there are indeed no quadratic divergencies, at least at one–loop order. It is easy to see that there are no quadratic divergencies from Yukawa interactions, since eqs.(7) hold. Eq.(7a) is satisfied because, as emphasized in the paragraph below eq.(35), each chiral superfield contains equal numbers of physical bosonic and fermionic degrees of freedom. Eq.(7b) can be checked by inserting the relevant part of the superportential $f_{\rm top} = \lambda_t T_L T_R H$, where T_L and T_R contain the left– and right–handed components of the top quark as well as the corresponding squarks, into eq.(39). In fact, eq.(39) even satisfies the more stringent requirements (10); the Yukawa contribution to $\pi_{\phi\phi}(0)$ therefore vanishes identically.[5]

We can now also compute the contribution from gauge interactions to the Higgs two–point function, using eqs.(40), (42) and (44). I will for the moment stick to the assumption that there is only one Higgs doublet; this will later prove to be not entirely realistic, but it is sufficient for the time being. As a further simplification, I will "switch off" hypercharge interactions, so that $m_W = m_Z$. Finally, I will use Feynman gauge, so contributions from the unphysical would–be Goldstone bosons have to be included; their mass is equal to m_W in this gauge.

[5] The alert reader may have noticed that the contribution (44) to the gauge interactions might produce contributions to m_f^2 that are proportional to m_W^2 or m_Z^2, thereby violating the condition (10a). We will come back to this point shortly.

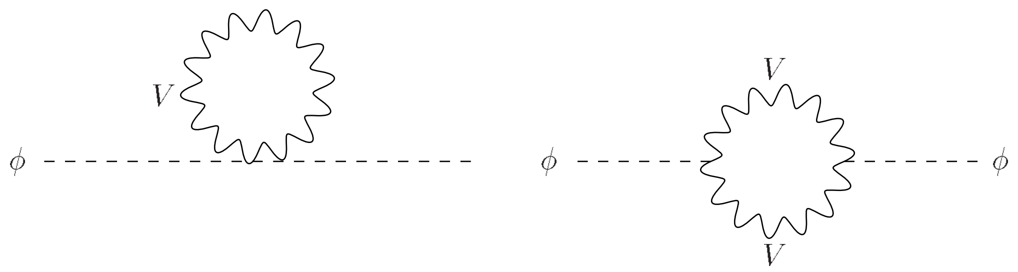

Fig. 5: Gauge boson loop contributions to the Higgs self energy. V stands for either $W^\pm$ or Z.

There are two types of contribution that involve only gauge bosons V in the loop, see Fig. 5:

$$\pi^{V}_{\phi\phi}(0) = N(V) \int \frac{d^4k}{(2\pi)^4} \frac{-ig_{\mu\nu}}{k^2 - m_W^2} i\frac{g^2}{2} g^{\mu\nu} = 3g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_W^2}. \tag{45}$$

$$\begin{aligned} \pi^{VV}_{\phi\phi}(0) &= N(V) \int \frac{d^4k}{(2\pi)^4} (ig_{\mu\nu} g m_W)(ig_{\rho\sigma} g m_W) \frac{(-ig^{\mu\rho})(-ig^{\nu\sigma})}{(k^2 - m_W^2)^2} \\ &= 6g^2 m_W \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_W^2)^2}. \end{aligned} \tag{46}$$

Here and in the subsequent expressions, the superscripts of $\pi_{\phi\phi}$ denote the particle(s) in the loop. Further, the effective number of vector bosons $N(V) = 3/2$, since Z boson loops get a suppression factor $1/2$ for identical particles.

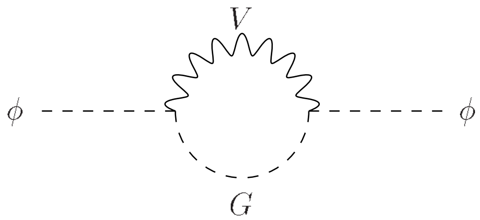

Fig. 6: Contributions to the Higgs self energy from loops involving a gauge bosons $V = W^\pm$ or Z and a would–be Goldstone boson $G = G^\mp$ or G^0.

There are also contributions with a gauge boson and a would–be Goldstone boson G in the loop, as shown in Fig. 6:

$$\pi^{ZG^0}_{\phi\phi}(0) = -\frac{g^2}{4} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_W^2)^2}; \tag{47a}$$

$$\pi^{W^\pm G^\mp}_{\phi\phi}(0) = -\frac{g^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_W^2)^2}. \tag{47b}$$

Note that eq.(47b) contains a factor of 2, since W^+G^- and W^-G^+ loops are distinct.

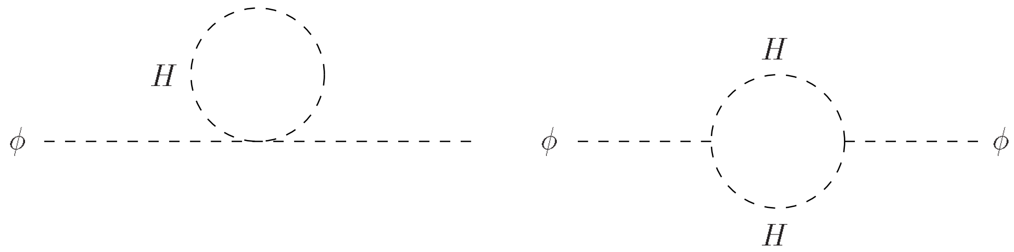

Fig. 7: Contributions to the Higgs self energy involving Higgs self interactions. H can be the physical Higgs field ϕ or one of the would–be Goldstone modes $G^\pm$ or G^0.

The last bosonic contributions come from Higgs self–interactions, see Fig. 7. It is important to note that in a supersymmetric model with only Higgs doublets it is *impossible* to introduce Higgs self–couplings through the superpotential f of eq.(36). Such an interaction would come from a cubic term in f, which is forbidden by gauge invariance. The only Higgs self–interactions therefore come from eq.(44). Focussing on the Higgs doublet field $H \equiv \left([\phi + v + iG^0]/\sqrt{2}, G^- \right)$, this term reads:

$$\begin{aligned} -V_D &= -\frac{1}{8} g^2 \left[\left(H_i^* \sigma_1^{ij} H_j \right)^2 + \left(H_i^* \sigma_2^{ij} H_j \right)^2 + \left(H_i^* \sigma_3^{ij} H_j \right)^2 \right] \\ &= -\frac{1}{8} g^2 \left[\frac{1}{2} (\phi + v)^2 + \frac{1}{2} \left(G^0 \right)^2 + \left| G^- \right|^2 \right]^2 ; \end{aligned} \tag{48}$$

recall that the properly normalized $SU(2)$ generators are $1/2$ times the Pauli matrices. From eq.(48) one finds the following contributions to the Higgs two–point function:

$$\pi_{\phi\phi}^{\phi}(0) = \frac{3}{8} g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\phi^2}; \tag{49a}$$

$$\pi_{\phi\phi}^{G^0}(0) = \frac{1}{8} g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_W^2}; \tag{49b}$$

$$\pi_{\phi\phi}^{G^\pm}(0) = \frac{1}{4} g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_W^2}. \tag{49c}$$

$$\pi_{\phi\phi}^{\phi\phi}(0) = \frac{9}{8} g^2 m_W^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k^2 - m_\phi^2 \right)^2}; \tag{50a}$$

$$\pi_{\phi\phi}^{G^0 G^0}(0) = \frac{1}{8} g^2 m_W^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_W^2)^2}; \tag{50b}$$

$$\pi_{\phi\phi}^{G^+ G^-}(0) = \frac{1}{4} g^2 m_W^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_W^2)^2}, \tag{50c}$$

where I have used $gv = 2m_W$; the contributions (49a,b) and (50a,b) again contain factors $1/2$ due to identical particle loops.

Clearly the bosonic contributions give a nonvanishing quadratic divergence, given by the sum of eqs.(45), (47) and (49). However, there are also fermionic contributions involving the higgsinos and gauginos. Their coupling to the Higgs field is determined by the next–to–last term in the second line of eq.(40), which couples the Higgs scalar to a gaugino and a higgsino. In the presence of gauge symmetry breaking, $v \neq 0$, this interaction also gives rise to a mass term between the gaugino and higgsino fields. Recall that the spinors in eq.(40) are two-component Weyl spinors. Such an off–diagonal mass term between Weyl spinors can be understood as a diagonal (Dirac) mass term for a four–component spinor that contains the two Weyl–spinors as its left– and right–handed components. In the present case this produces a charged "chargino" $\widetilde{W}$ with mass $\sqrt{2}m_W$, as well as a (Dirac) "neutralino" $\tilde{Z}$ with mass m_W. (Recall that hypercharge interactions are switched off for the time being.) The chargino field couples to the Higgs boson ϕ with strength $g/\sqrt{2}$, while the neutralino couples with strength $g/2$. This relative factor of $\sqrt{2}$ between the mass and coupling of the chargino and those of the neutralino follows from the fact that the charged $SU(2)$ gauginos are given by $(\lambda_1 \pm i\lambda_2)/\sqrt{2}$, while the neutral gaugino is simply λ_3. This gives the following fermionic contributions, see eq.(4):

$$\pi_{\phi\phi}^{\widetilde{W}\widetilde{W}}(0) = -2g^2 \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2 - 2m_W^2} + \frac{4m_W^2}{\left(k^2 - 2m_W^2\right)^2} \right]; \tag{51a}$$

$$\pi_{\phi\phi}^{\tilde{Z}\tilde{Z}}(0) = -g^2 \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2 - m_W^2} + \frac{2m_W^2}{\left(k^2 - m_W^2\right)^2} \right]. \tag{51b}$$

We see that the total quadratic divergence does indeed cancel! The sum of eqs.(51) cancels the contribution (45), while the contribution from eqs.(47) cancels that from eqs.(49). The contributions from eqs.(46) and (50) are by themselves only logarithmically divergent.

Before breaking out the champagne, we should check that we have not missed any terms – and, indeed, we have! The scalar self–interaction (44) also contains terms

$$\mathcal{L}_{\phi\tilde{f}} = \frac{g^2}{2} \left(\frac{1}{2}\phi^2 + v\phi \right) \sum_f I_{3,f} \left| \tilde{f} \right|^2, \tag{52}$$

which leads to a total contribution from sfermion tadpole diagrams:

$$\pi_{\phi\phi}^{\tilde{f}}(0) = \frac{g^2}{2} \sum_f I_{3,f} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{\tilde{f}}^2}. \tag{53}$$

Here, $I_{3,f}$ is the weak isospin of fermion f (which is identical to that of its bosonic superpartner $\tilde{f}$, of course). Fortunately the trace of I_3 over a complete representation of $SU(2)$ vanishes, so the contribution (53) is in fact not quadratically divergent. Consider the case of a single $SU(2)$ doublet. Let m_+ be the mass of of the $I_3 = +1/2$ sfermion. Gauge invariance implies that the $I_3 = -1/2$ sfermion must have the same mass, except for the contributions from eq.(44) due to the spontaneous breaking of $SU(2)$. The mass of the $I_3 = -1/2$ sfermion is then in total given by $m_+^2 - m_W^2$, and the contribution to eq.(53) becomes:

$$\begin{aligned} \pi_{\phi\phi}^{\tilde{f}}(0)\Big|_{1 \text{ doublet}} &= \frac{g^2}{4} \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 - m_+^2} - \frac{1}{k^2 - m_+^2 + m_W^2} \right) \\ &= \frac{g^2}{4} \int \frac{d^4k}{(2\pi)^4} \frac{m_W^2}{\left(k^2 - m_+^2\right)\left(k^2 - m_+^2 + m_W^2\right)}, \end{aligned} \tag{54}$$

which is only logarithmically divergent.

So far, so good. We are clearly on the right track: There is no one–loop quadratic divergence from $SU(2)$ interactions. This proof can be extended to all orders using "supergraphs", i.e. Feynman rules for superfields [6]. However, disaster strikes when we try to re–introduce hypercharge interactions. Much of the calculation presented here still goes through, with minor changes. However, at the end a nonvanishing divergence remains, if the model only contains one Higgs doublet and any number of complete (s)fermion generations of the SM. The problem can be traced back to the fact that in this case the total trace of the hypercharge generator does not vanish. Its trace over a complete (s)fermion generation does vanish, but this leaves the contribution from the single Higgs doublet. Clearly the model we have been using in this subsection is still not fully realistic.[6]

Finally, notice that even in the absence of hypercharge interactions the total gauge contribution to $\pi_{\phi\phi}(0)$ does not vanish; in fact, some logarithmic divergencies remain. This is related to the fact that in the model considered here, one cannot break $SU(2) \times U(1)_Y$ invariance without breaking supersymmetry. This brings us to the issue of supersymmetry breaking, to which we turn next.

3f. Supersymmetry Breaking

As emphasized earlier, the supersymmetric Lagrangian constructed in Sec. 3d satisfies eq.(10a), that is, the masses of the "ordinary" SM particles and their superpartners are identical. This is clearly not realistic; there is no selectron with mass 511 keV, nor is there a smuon with mass 106 MeV, etc. Indeed, as mentioned in the Introduction, no superpartners have been discovered yet. Searches at the e^+e^- collider LEP imply that all charged sparticles must be heavier than 60 to 80 GeV [13]. Similarly, searches at the Tevatron $p\bar{p}$ collider imply bounds on squark and gluino masses between 150 and 220 GeV [14]. Hence supersymmetry must be broken.

The great success of the Standard Model with its broken $SU(2) \times U(1)_Y$ symmetry should have convinced everyone of the usefulness of broken symmetries. Unfortunately it is not easy to break supersymmetry spontaneously. One problem follows directly from the definition of the SUSY algebra, eq.(12b), which implies

$$\frac{1}{4}\left(\overline{Q}_1 Q_1 + Q_1 \overline{Q}_1 + \overline{Q}_2 Q_2 + Q_2 \overline{Q}_2\right) = P^0 \equiv H \geq 0, \tag{55}$$

where H is the Hamiltonian (energy operator). The fact that this is non–negative simply follows from it being a sum of perfect squares. If the vacuum state $|0\rangle$ is supersymmetric, then $Q_\alpha|0\rangle = \overline{Q}_{\dot\alpha}|0\rangle = 0$, and eq.(54) implies $E_{\rm vac} \equiv \langle 0|H|0\rangle = 0$. On the other hand, if the vacuum state is *not* supersymmetric, i.e. at least one SUSY generator does not annihilate the vacuum, then eq.(55) implies $E_{\rm vac} > 0$. In other words, global supersymmetry can only be broken spontaneously if there is a positive vacuum energy. This might give rise to a troublesome cosmological constant, although

[6] Hypercharge loop contributions to the Higgs mass have another peculiarity. As already mentioned, the trace of the hypercharge over a complete generation vanishes, given an only logarithmically divergent $\tilde{f}$ contribution in analogy with eq.(53). However, at least in most models the masses of different sfermions within the same generation are not related by gauge invariance, so there could in general be (very) large mass splittings. The scale of the log–divergent and finite contributions to $\pi_{\phi\phi}$ would then be set by these mass splittings, not by m_W^2. This becomes a concern if one tries to push the masses of the sfermions of the first two generations to very large values [11, 12], which otherwise need not lead to unacceptably large corrections to $\pi_{\phi\phi}$, due to the smallness of the first and second generation Yukawa couplings. Finally, in the context of supergravity or superstring theory, an anomalous $U(1)$ factor can be part of a consistent and phenomenologically acceptable model; see e.g. the first ref.[12].

the connection between a microscopic vacuum energy and a macroscopic cosmological constant is not entirely straightforward [15].

As an example of the general result (55), consider the case where supersymmetry is broken by the vev of some scalar particle, in direct analogy to $SU(2) \times U(1)_Y$ breaking in the SM. The scalar potential contains two pieces, given in eqs.(39) and (44):

$$V = \sum_i \left| \frac{\partial f}{\partial \phi_i} \right|^2 + \sum_l \frac{g_l^2}{2} \sum_a \left| \sum_{i,j} \phi_i^* T_{l,a}^{ij} \phi_j \right|^2 , \tag{56}$$

where l labels the simple groups whose product forms the entire gauge group of the model (e.g., $SU(3) \times SU(2) \times U(1)_Y$ in the SM). We see that indeed $V \geq 0$. We can therefore break SUSY if either $\langle F_i \rangle = \langle \partial f / \partial \phi_i \rangle \neq 0$ for some i ["F–term breaking", see eq.(38)], or if $\langle D_{l,a} \rangle = \langle \sum_{i,j} \phi_i^* T_{l,a}^{ij} \phi_j \rangle \neq 0$ for some combination (l, a) [("D–term breaking", see eq.(43)]; in the latter case some gauge symmetries will be broken as well. In fact, the example we discussed in the previous subsection has D–term breaking, since the D–term associated with the I_3 generator has a nonvanishing vev, see eq.(48); this explains why the total contribution to $\pi_{\phi\phi}$ did no vanish in this example. However, clearly the second term in eq.(56) can be minimized (set to zero) if all vevs vanish, $\langle \phi_i \rangle = 0$ for all i. Turning the symmetry breaking point into the absolute minimum of the potential therefore requires nontrivial contributions from the first term in eq.(56).

The construction of realistic models with spontaneously broken SUSY is made even more difficult by the fact that in such models eq.(10a) still remains satisfied "on average". More exactly, the supertrace over the whole mass matrix vanishes in more with pure F–term breaking [16]:

$$\mathrm{Str}\mathcal{M}^2 \equiv \sum_J (-1)^{2J} \mathrm{tr}\mathcal{M}_J^2 = 0, \tag{57}$$

where J is the spin, and $\mathcal{M}_J$ is the mass matrix for all particles with spin J. This is problematic, because we want *all* sfermions to be significantly heavier than their SM partners (with the possible exception of the scalar top). In principle one could still satisfy the constraint (57) by making the gauginos quite heavy; unfortunately this seems almost impossible to achieve in practice. All potentially realistic globally supersymmetric models of spontaneous SUSY breaking where sparticles get masses at tree–level therefore contain a new $U(1)$ whose D–term is nonzero in the minimum of the potential, as well as a rather large number of superfields beyond those required by the field content of the SM [7]. Recent models that attempt to break global SUSY spontaneously instead circumvent the constraint (57) by creating most sparticle masses only through radiative corrections [17]; this also necessitates the introduction of several additional superfields.

Most phenomenological analyses therefore do not attempt to understand SUSY breaking dynamically; rather, it is parametrized by simply inserting "soft breaking terms" into the Lagrangian. "Soft" here means that we want to maintain the cancellation of quadratic divergencies; e.g. we want to respect eqs.(7). The explicit calculation of Sec. 2 showed that, at least to one–loop order, quadratic divergencies still cancel even if we introduce

- scalar mass terms $-m_{\phi_i}^2 |\phi_i|^2$, and
- trilinear scalar interactions $-A_{ijk}\phi_i\phi_j\phi_k + h.c.$

into the Lagrangian. Girardello and Grisaru [18] have shown that this result survives in all orders in perturbation theory. They also identified three additional types of soft breaking terms:

- gaugino mass terms $-\frac{1}{2}m_l\bar{\lambda}_l\lambda_l$, where l again labels the group factor;
- bilinear terms $-B_{ij}\phi_i\phi_j + h.c.$; and
- linear terms $-C_i\phi_i$.

Of course, linear terms are gauge invariant only for gauge singlet fields.[7] Note that we are *not* allowed to introduce additional masses for chiral fermions, beyond those contained in the superpotential. Also, the relations between dimensionless couplings imposed by supersymmetry must not be broken.

This completes our discussion of the construction of "realistic" supersymmetric field theories. Let us now apply these results to the simplest such model.

4. The Minimal Supersymmetric Standard Model

Let us now try to construct a fully realistic SUSY model, i.e. a theory with softly broken supersymmetry that satisfies all phenomenological constraints. As already emphasized repeatedly, the main motivation for introducing weak–scale supersymmetry is the absence of quadratic divergencies, which leads to a solution of the (technical aspect of the) hierarchy problem. There are, however, further arguments why supersymmetric theories might be interesting. One is based on the Haag–Lopuszanski–Sohnius (HLS) theorem [10]; it states that the biggest symmetry which an interacting, unitary field theory can have is the direct product of a (possibly very large) gauge symmetry, Lorentz invariance, and (possibly extended) supersymmetry. The first two ingredients are part of the highly succesful Standard Model; this naturally raises the question whether making use of the third kind of symmetry allowed by the HLS theorem leads to an even better description of Nature.

Furthermore, supersymmetry appears very naturally in superstring theory. Often the existence of space–time supersymmetry is even considered to be a firm prediction of string theory. String theory, in turn, is clearly our currently best hope for a "theory of everything", which would, in particular, include a quantum theory of gravity. However, this argument only requires supersymmetry at or below the Planck scale, not necessarily at the weak scale.

These two arguments are admittedly rather speculative. A more practical advantage of supersymmetric theories becomes apparent when we compare them with their main competitor, technicolor models [20]. In these models one tries to solve the problem of quadratic divergencies by dispensing with elementary scalars altogether. The Higgs mechanism is then replaced by a non–perturbative mechanism, where a confined "technicolor" gauge interaction leads to the formation of "techniquark" condensates, which break (local) $SU(2) \times U(1)_Y$ invariance in a way reminiscent of the breaking of the (global) chiral symmetry of QCD by quark condensates. I personally find the Higgs mechanism much more elegant and innovative, but many of my colleagues seem to consider the technicolor idea to be at least in principle more appealing, since it appears to give a more dynamical understanding of gauge symmetry breaking. In practice this hope is not really borne out, however: Since gauge symmetry breaking is assumed to be due to some non–perturbative dynamics, it is very difficult to make firm predictions for physical observables. The lessons learned from the study of low–energy hadron physics unfortunately turned out to be rather useless here, since a successful technicolor theory must not be a scaled–up version of QCD; such a theory would

[7] Under certain circumstances one can also introduce trilinear interactions of the form $\tilde{A}_{ijk}\phi_i\phi_j\phi_k^* + h.c.$ [19].

have much too strong flavor changing neutral currents, and give too large contributions to certain electroweak precision variables, most notably the "S–parameter" [21].

In contrast, supersymmetric theories might allow a perturbative description of Nature at energies between about 1 GeV and the Planck scale. Furthermore, it is quite easy to construct a potentially realistic SUSY model, as will be demonstrated in the subsequent subsection.

4a. Definition of the Model

As implied by the name, the minimal supersymmetric standard model (MSSM) is essentially a straightforward supersymmetrization of the SM. In particular, "minimal" means that we want to keep the number of superfields and interactions as small as possible. Since the SM matter fermions reside in different representations of the gauge group than the gauge bosons, we have to place them in different superfields; no SM fermion can be identified as a gaugino.[8] One generation of the SM is therefore described by five left–chiral superfields: Q contains the quark and squark $SU(2)$ doublets, U^c and D^c contains the (s)quark singlets, L contains the (s)lepton doublets, and E^c contains the (s)lepton singlets. Note that the $SU(2)$ singlet superfields contain left–handed anti–fermions; their scalar members therefore have charge $+1$ for $\tilde{e}_R^c$, $-2/3$ for $\tilde{u}_R^c$, and $+1/3$ for $\tilde{d}_R^c$. Of course, we need three generations to describe the matter content of the SM.

As discussed in Sec. 3, we have to introduce vector superfields to describe the gauge sector. In particular, we need eight gluinos $\tilde{g}$ as partners of the eight gluons of QCD, three winos $\widetilde{W}$ as partners of the $SU(2)$ gauge bosons, and a bino $\widetilde{B}$ as $U(1)_Y$ gaugino. Since $SU(2) \times U(1)_Y$ is broken, the winos and the bino are in general not mass eigenstates; rather, they mix with fields with the same charge but different $SU(2) \times U(1)_Y$ quantum numbers. We will come back to this point in Sec. 4c.

The only slight subtlety in the field content of the MSSM is in the choice of the Higgs sector. As in the SM, we want to break $SU(2) \times U(1)_Y$ invariance by $SU(2)$ doublet scalars with hypercharge $|Y| = 1/2$. Looking through the fields that have already been introduced, one notices that the slepton doublets $\tilde{l}_L$ fulfill this requirement. It is thus natural to ask whether the sneutrino fields can play the role of the Higgs boson of the SM. Unfortunately the answer is No [22]. The terms required to give masses to the charged leptons explicitly break lepton number, if the sneutrinos were to serve as Higgs fields. This leads to a host of problems. The most stringent bound on doublet sneutrino vevs comes from the requirement that all neutrino masses must be very small [23], which implies $\langle \tilde{\nu} \rangle^2 \ll M_Z^2$ [24].

We therefore have to introduce dedicated Higgs superfields to break $SU(2) \times U(1)_Y$. Indeed, we need at least two such superfields: H has hypercharge $Y = -1/2$, while $\bar{H}$ has $Y = +1/2$. There are at least three reasons for this. First, we saw in Sec. 3e that a model with a single Higgs doublet superfield suffers from quadratic divergencies, since the trace of the hypercharge generator does not vanish. This already hints at the second problem: A model with a single Higgs doublet superfield has nonvanishing gauge anomalies associated with fermion triangle diagrams. The contribution from a complete generation of SM fermions does vanish, of course, since the SM is anomaly–free. However, if we only add a single higgsino doublet, anomalies will be introduced; we need a second higgsino doublet with opposite hypercharge to cancel the contribution from the first doublet.

[8] One occasionally sees the statement that supersymmetry links gauge and matter fields. This is not true in the MSSM, nor in any potentially realistic SUSY model I know.

Finally, as discussed in Sec. 3f, the masses of chiral fermions must be supersymmetric, i.e. they must originate from terms in the superpotential. On the other hand, we have seen in Sec. 3d that the superpotential must not contain products of left–chiral and right–chiral superfields. This means that we are not allowed to introduce the hermitean conjugate of a Higgs superfield (or of any other chiral superfield) in f. It would then be impossible to introduce $U(1)_Y$ invariant terms that give masses to both up–type and down–type quarks if there is only one Higgs superfield; we again need (at least) two doublets.

Having specified the field content of the MSSM, we have to define the interactions. Of course, the gauge interactions are determined uniquely by the choice of gauge group, which we take to be $SU(3) \times SU(2) \times U(1)_Y$ as in the SM. However, while the gauge symmetries constrain the superpotential f, they do not fix it completely. We can therefore appeal to a principle of minimality and only introduce those terms in f that are *necessary* to build a realistic model. Alternatively, we can demand that f respects lepton and baryon number; these are automatic ("accidental") symmetries of the SM, but could easily be broken explicitly in the MSSM, as we will see below. Either approach leads to the following superpotential:

$$f_{\rm MSSM} = \sum_{i,j=1}^{3} \left[(\lambda_E)_{ij} \, H L_i E_j^c + (\lambda_D)_{ij} \, H Q_i D_j^c + (\lambda_U)_{ij} \, \bar{H} Q_i U_j^c \right] + \mu H \bar{H}. \tag{58}$$

Here i and j are generation indices, and contractions over $SU(2)$ and $SU(3)$ indices are understood. For example,

$$H\bar{H} \equiv H_1 \bar{H}_2 - H_2 \bar{H}_1; \tag{59a}$$

$$Q D_R^c \equiv \sum_{n=1}^{3} Q_n \left(D_R^c \right)_n . \tag{59b}$$

The matrices λ_D and λ_U give rise to quark masses and to the mixing between quark current eigenstates as described by the familiar KM matrix [23]. Since the superpotential (58) leaves neutrinos exactly massless, as in the SM, the matrix λ_E can be taken to be diagonal.

The choice of the superpotential (58) leads to "R parity conservation". Note that the gauge interactions described by eqs.(40), (42) and (42) only introduce terms in the Lagrangian that contain an even number of superpartners (gauginos, sfermions or higgsinos). For example, if Φ is a matter superfield, the first term in eq.(40) has two sparticles (sfermions); the second, none; the third, four (sfermions, after eq.(43) has been used); and the fourth, two (one sfermion and one gaugino). If Φ is a Higgs superfield, the first term in eq.(40) contains no sparticles; the second, two (higgsinos); the third, none; and the fourth, two (one higgsino and one gaugino). The fact that gauge interactions always involve an even number of sparticles implies that they respect an "R parity", under which all SM fields (matter fermions, and Higgs and gauge bosons) are even while all sparticles (sfermions, higgsinos and gauginos) are odd.

The interactions produced by the superpotential (58) also respect this symmetry; this can easily be verified by plugging it into eq.(39). This means that in the MSSM one has to produce sparticles in pairs, if one starts with beams of ordinary particles. For example, one can produce a pair of sleptons from the decay of a (virtual) Z boson using the first term in eq.(40). Since we saw in Sec. 3f that sparticles have to be quite heavy, this constraint reduces the "mass reach" of a given collider for sparticle searches. For example, at e^+e^- colliders one can generally only produce sparticles with mass below the beam energy, which is only half the total center–of–mass energy.

Furthermore, a sparticle can only decay into an odd number of other sparticles and any number of SM particles. For example, a squark might decay into a quark and a higgsino via a Yukawa interaction described by the first term of eq.(39), if this decay is kinematically allowed. In the MSSM the lightest supersymmetric particle (LSP) therefore cannot decay at all; it is absolutely stable. This gives rise to characteristic signatures for sparticle production events at colliders, which allow to distinguish such events from "ordinary" SM events [25]. The argument goes as follows. Since LSPs are stable, some of them must have survived from the Big Bang era. If LSPs had strong or electromagnetic interactions, many or most of these cosmological relics would have bound to nuclei. Since the LSPs would have to be quite massive in such scenarios, this would give rise to "exotic isotopes", nuclei with very strange mass to charge ratios. Searches [26] for such exotics have led to very stringent bounds on their abundance, which exclude all models with stable charged or strongly interacting particles unless their mass exceeds several TeV [27]. In the context of the MSSM this means that the LSP must be neutral. As far as collider experiments are concerned, an LSP will then look like a heavy neutrino, that is, it will not be detected at all, and will carry away some energy and momentum. Since *all* sparticles will rapidly decay into (at least) one LSP and any number of SM particles, the MSSM predicts that *each* SUSY event has some "missing (transverse) energy/momentum".

Note that this property is not an automatic consequence of our choice of field content and gauge group. We could have introduced the following terms in the superpotential which explicitly break R parity:

$$f_{\text{R-breaking}} = \lambda L L E^c + \lambda' L Q D^c + \lambda'' D^c D^c U^c + \mu' H L, \tag{60}$$

where generation indices have been suppressed. The first two terms in eq.(60) break lepton number L, while the third terms break baryon number B. Within the MSSM field content one can therefore break R parity only if either L or B are not conserved; however, in general SUSY models this relation between B and L conservation on the one hand and R parity on the other does not hold. If both B and L were broken, the proton would decay very rapidly; at least some of the couplings in eq.(60) therefore have to be zero (or very, very small). This makes it very difficult to embed the MSSM into some Grand Unified model, unless *all* the couplings in eq.(60) are (almost) zero, which we will assume from now on. See refs.[25] and [28] for further discussions of the theory and phenomenology of models where R parity is broken.

So far we have only specified the supersymmetry conserving part of the Lagrangian. In the gauge and Yukawa sectors we have had to introduce the same number of free parameters as in the SM; in the Higgs sector the single parameter μ replaces two parameters of the SM. However, in general we have to introduce a very large number of free parameters to describe SUSY breaking, as discussed in Sec. 3f:

$$\begin{aligned} -\mathcal{L}_{\text{MSSM, non-SUSY}} &= m^2_{\tilde q} \left|\tilde q_L\right|^2 + m^2_{\tilde u} \left|\tilde u^c_R\right|^2 + m^2_{\tilde d} \left|\tilde d^c_R\right|^2 + m^2_{\tilde l} \left|\tilde l_L\right|^2 + m^2_{\tilde e} \left|\tilde e^c_R\right|^2 \\ &+ \left(\lambda_E A_E H \tilde l_L \tilde e^c_R + \lambda_D A_D H \tilde q_L \tilde d^c_R + \lambda_U A_U \bar H \tilde q_L \tilde u^c_R + B\mu H \bar H + h.c.\right) \\ &+ m^2_H \left|H\right|^2 + m^2_{\bar H} \left|\bar H\right|^2 + \frac{1}{2} M_1 \widetilde B \widetilde B + \frac{1}{2} M_2 \widetilde W \widetilde W + \frac{1}{2} M_3 \widetilde g \widetilde g. \end{aligned} \tag{61}$$

Here I have used the same symbols for the Higgs scalars H, $\bar H$ as for the corresponding superfields. Note that $m^2_{\tilde q}$, $m^2_{\tilde u}$, $m^2_{\tilde d}$, $m^2_{\tilde l}$ and $m^2_{\tilde e}$ are in general hermitean 3×3 matrices, while $\lambda_U A_U$, $\lambda_D A_D$ and $\lambda_E A_E$ are general 3×3 matrices. If we allow these parameters to be complex, the SUSY breaking piece (61) of the Lagrangian contains more than 100 unknown real constants! Fortunately most processes will be sensitive only to some (small) subset of these parameters, at least at tree level. Finally, note that eq.(61) also respects R parity. Introducing R parity breaking terms like $\tilde{l}_L \tilde{l}_L \tilde{e}_R^c$ would lead to an unstable vacuum, i.e. the scalar potential would be unbounded from below, unless we also introduce the corresponding terms in the superpotential (60).

This completes the definition of the MSSM. We are now ready to investigate some of its properties in more detail.

4b. Electroweak Symmetry Breaking in the MSSM

Given that the (still hypothetical) existence of elementary Higgs bosons leads to the main motivation for the introduction of weak–scale supersymmetry, it seems reasonable to start the discussion of the phenomenology of the MSSM with a treatment of its Higgs sector. This will also lead to a very strong and in principle readily testable prediction.

Of course, one wants $SU(2) \times U(1)_Y$ to be broken spontaneously, i.e. the scalar potential should have its absolute minimum away from the origin. Let us for the moment focus on the part of the potential that only depends on the Higgs fields. It receives three types of contributions: Supersymmetric "F–terms" from the last term in eq.(39) only contribute mass terms $\mu^2 \left(|H|^2 + |\bar{H}|^2 \right)$; supersymmetric "D–terms", eq.(44), give rise to quartic interactions; and the SUSY breaking part (61) of the Lagrangian gives additional mass and mixing terms. Altogether, one finds:

$$\begin{aligned} V_{\rm Higgs} &= m_1^2 \left| H \right|^2 + m_2^2 \left| \bar{H} \right|^2 + \left(m_3^2 H \bar{H} + h.c. \right) \\ &+ \frac{g_1^2 + g_2^2}{8} \left(\left| H^0 \right|^2 - \left| \bar{H}^0 \right|^2 \right)^2 + \left(\text{D} - \text{terms for } H^-,\ \bar{H}^+ \right), \end{aligned} \tag{62}$$

where g_1 and g_2 are the $U(1)_Y$ and $SU(2)$ gauge couplings, and the mass parameters are given by

$$m_1^2 = m_H^2 + \mu^2; \tag{63a}$$

$$m_2^2 = m_{\bar{H}}^2 + \mu^2; \tag{63b}$$

$$m_3^2 = B \cdot \mu. \tag{63c}$$

One first has to check that one can still choose the vacuum expectation values such that charge is conserved in the absolute minimum of the potential (62). This is indeed the case. By using $SU(2)$ gauge transformations, one can (e.g.) chose $\langle H^- \rangle = 0$, without loss of generality. The derivative $\partial V_{\rm Higgs} / \partial H^-$ can then only be made to vanish if $\langle \bar{H}^+ \rangle = 0$ as well.[9] We can therefore ignore the charged components H^-, $\bar{H}^+$ when minimizing the potential.[10] Furthermore, $v \equiv \langle H^0 \rangle$ and $\bar{v} \equiv \langle \bar{H}^0 \rangle$ can be chosen to be real. The only contribution to the potential (62) that is sensitive to the complex phases of the fields is the term $m_3^2 H^0 \bar{H}^0 + h.c$, which (for real m_3^2) is minimized if $\text{sign}(v\bar{v}) = -\text{sign}(m_3^2)$. This means that CP invariance cannot be broken spontaneously in the MSSM.

[9] This equation has a second solution, $g^2 H^{0*} \bar{H}^{0*} = 2m_3^2$; however, it is easy to see that this does not correspond to a minimum of the potential.

[10] Even though the Higgs sector conserves charge, it might still be broken in the absolute minimum of the complete potential, where some sfermions may have nonzero vev. See ref.[29] for a detailed discussion of this point.

Note that the strength of the quartic interactions is determined by the gauge couplings here; in contrast, in the nonsupersymmetric SM the strength of the Higgs self–interaction is an unknown free parameter. Moreover, in the direction $|H^0| = \left|\bar{H}^0\right|$, the quartic term in (62) vanishes identically. The potential is therefore only bounded from below if

$$m_1^2 + m_2^2 \geq 2\left|m_3^2\right|. \tag{64}$$

This condition implies that m_1^2 and m_2^2 cannot both be negative. Nevertheless we can still ensure that the origin of the potential is only a saddle point, i.e. that $SU(2) \times U(1)_Y$ is broken in the minimum of the potential, by demanding that the determinant of second derivatives of the potential (62) at the origin is negative, which requires

$$m_1^2 m_2^2 < m_3^4. \tag{65}$$

It is important to note that the conditions (64) and (65) cannot be satisfied simultaneously if $m_1^2 = m_2^2$. Further, eqs.(63a,b) show that the supersymmetric contribution to m_1^2 and m_2^2 is the same; any difference between these two quantities must be due to the SUSY breaking contributions m_H^2 and $m_{\bar{H}}^2$. In other words, in the MSSM there is an intimate connection between gauge symmetry breaking and SUSY breaking: The former is *not possible* without the latter.

The Higgs potential can now be minimized straightforwardly by solving the equations $\partial V_{\rm Higgs}/\partial H^0 = \partial V_{\rm Higgs}/\partial \bar{H}^0 = 0$. Usually it is most convenient to solve these equations for some of the parameters in eq.(62), rather than for the vevs v and $\bar{v}$. The reason is that the combination of vevs

$$\frac{g_1^2 + g_2^2}{2}\left(v^2 + \bar{v}^2\right) = M_Z^2 = (91.18 \text{ GeV})^2 \tag{66}$$

is very well known. We can therefore describe both vevs in terms of a single parameter,

$$\tan\beta \equiv \bar{v}/v. \tag{67}$$

The minimization conditions can then be written as

$$m_1^2 = -m_3^2 \tan\beta - \frac{1}{2} M_Z^2 \cos(2\beta); \tag{68a}$$

$$m_2^2 = -m_3^2 \cot\beta + \frac{1}{2} M_Z^2 \cos(2\beta). \tag{68b}$$

This form is most convenient for the calculation of the Higgs mass matrices described below. Alternatively, one can use eqs.(63) to derive

$$B \cdot \mu = \frac{1}{2}\left[\left(m_H^2 - m_{\bar{H}}^2\right)\tan(2\beta) + M_Z^2 \sin(2\beta)\right]; \tag{69a}$$

$$\mu^2 = \frac{m_{\bar{H}}^2 \sin^2\beta - m_H^2 \cos^2\beta}{\cos(2\beta)} - \frac{1}{2} M_Z^2. \tag{69b}$$

This form is most convenient if one has some (predictive) ansatz for the soft SUSY breaking terms; one such example will be discussed in Sec. 4d.

After symmetry breaking, three of the eight degrees of freedom contained in the two complex doublets H and $\bar{H}$ get "eaten" by the longitudinal modes of the $W^\pm$ and Z gauge bosons. The five physical degrees of freedom that remain form a neutral pseudoscalar Higgs bosons A_p, two neutral scalars h_p and H_p, and a charged Higgs boson $H_p^\pm$, where the subscript p stands for "physical"; this is to be compared with the single physical neutral scalar Higgs boson of the SM. The tree–level mass matrices for these Higgs states can most easily be computed from the matrix of second derivatives of the Higgs potential (62), taken at its absolute minimum. The physical pseudoscalar Higgs boson A_p is made from the imaginary parts of H^0 and $\bar{H}^0$, which have the mass matrix [in the basis $(\Im H^0/\sqrt{2}, \Im \bar{H}^0/\sqrt{2})$]:

$$\mathcal{M}_I^2 = \begin{pmatrix} -m_3^2 \tan\beta & -m_3^2 \\ -m_3^2 & -m_3^2 \cot\beta \end{pmatrix}, \tag{70}$$

where I have used eqs.(68). Note that $\det \mathcal{M}_I^2 = 0$; the corresponding massless mode is nothing but the neutral would–be Goldstone boson $G^0 = \frac{1}{\sqrt{2}}\left(\cos\beta \Im H^0 - \sin\beta \Im \bar{H}^0\right)$. The physical pseudoscalar is orthogonal to G^0: $A_p = \frac{1}{\sqrt{2}}\left(\sin\beta \Im H^0 + \cos\beta \Im \bar{H}^0\right)$, with mass

$$m_A^2 = \mathrm{tr}\mathcal{M}_I^2 = -\frac{2m_3^2}{\sin(2\beta)}. \tag{71}$$

[Recall that $\mathrm{sign}(v\bar{v}) \equiv \mathrm{sign}(\sin 2\beta) = -\mathrm{sign}(m_3^2)$.]

Note that $m_A^2 \to 0$ as $m_3^2 \to 0$. Such a massless pseudoscalar "axion" is excluded experimentally (if it is connected to $SU(2) \times U(1)_Y$ breaking; other, "invisible" axions are still allowed [30]). This massless state occurs since for $m_3^2 = 0$, the Higgs potential (62) is invariant under an additional global $U(1)$, where both H and $\bar{H}$ have the same charge. This new global symmetry is also broken by the vevs, giving rise to an additional Goldstone boson. The m_3^2 term breaks this symmetry explicitly, thereby avoiding the existence of an axion. Furthermore, eq.(69a) shows that $m_3^2 = 0$ implies $\sin(2\beta) = 0$, i.e. $v \cdot \bar{v} = 0$. This means that a nonvanishing m_3^2 is also necessary to give vevs to both Higgs bosons, which in turn are needed to give masses to both up–type and down–type quarks, as can be seen from eq.(58).

This causes an (aesthetic) problem. We have seen above that in the MSSM, $SU(2) \times U(1)_Y$ breaking requires SUSY breaking. Now we find that we can break $SU(2) \times U(1)_Y$ in a phenomenologically acceptable way only if $m_3^2 \neq 0$, which implies $\mu \neq 0$, see eq.(63c). This means that we need to introduce mass parameters both in the supersymmetry breaking and in the supersymmetry conserving parts of the Lagrangian. Moreover, the two kinds of dimensionful parameters must be of roughly the same order of magnitude. Such a connection between these two sectors of the theory is, at this level at least, quite mysterious. However, several solutions to this "$\mu-$problem" have been suggested; see ref.[28] for a further discussion of this point.

The neutral scalar Higgs bosons are mixtures of the real parts of H^0 and $\bar{H}^0$. The relevant mass matrix is in the basis $(\Re H^0/\sqrt{2}, \Re \bar{H}^0/\sqrt{2})$:

$$\mathcal{M}_R^2 = \begin{pmatrix} -m_3^2 \tan\beta + M_Z^2 \cos^2\beta & m_3^2 - \frac{1}{2} M_Z^2 \sin(2\beta) \\ m_3^2 - \frac{1}{2} M_Z^2 \sin(2\beta) & -m_3^2 \cot\beta + M_Z^2 \sin^2\beta \end{pmatrix}. \tag{72}$$

Note that $\det \mathcal{M}_R^2 = m_A^2 M_Z^2 \cos^2(2\beta)$ goes to zero if either $m_A \to 0$, or $M_Z \to 0$, or $\tan\beta \to 1$ [which implies $\cos(2\beta) \to 0$]. These three different limits therefore all lead to the existence of a massless Higgs boson (at least at tree–level). In general, the eigenvalues of $\mathcal{M}_R^2$ are given by:

$$m_{H,h}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 \pm \sqrt{\left(m_A^2 + M_Z^2\right)^2 - 4m_A^2 M_Z^2 \cos^2(2\beta)}\right]. \tag{73}$$

This leads to the important upper bound [31]

$$m_h \leq \min(m_A, M_Z) \cdot |\cos(2\beta)|. \tag{74}$$

The MSSM seems to predict that one of the neutral Higgs scalars must be lighter than the Z boson! The origin of this strong bound can be traced back to the fact that the only Higgs self couplings in eq.(62) are electroweak gauge couplings. In contrast, in the nonsupersymmetric SM the strength of this coupling is unknown, and no comparable bound on the Higgs mass can be derived.

Unfortunately the bound (74) receives radiative corrections already at the 1–loop level, the dominant contribution coming from top–stop loops [32]. These become large if stop masses are significantly bigger than m_t. At scales between the stop and top masses, the Higgs sector should more properly be described as in the non–supersymmetric SM, where the top Yukawa coupling gives a sizable correction to the quartic Higgs self coupling, and hence to the Higgs mass. In leading logarithmic approximation the bound (74) is then modified to

$$m_h^2 \leq M_Z^2 \cos^2(2\beta) + \frac{3m_t^4}{32\pi^2 \sin^2\beta M_W^2} \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}, \tag{75}$$

where $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are the masses of the two stop eigenstates (see Sec. 4c). Numerically this gives

$$m_h \leq 130 \text{ GeV}, \tag{76}$$

if one assumes that stop masses do not exceed 1 TeV significantly, and uses the bound[11] $m_t < 185$ GeV; the upper limit in (76) includes a (rather generous) contribution of about 10 GeV from non–logarithmic corrections [33].[12]

The prediction (76) is in principle quite easily testable. If the bound (75) is (nearly) saturated, h_p becomes very similar to the Higgs boson of the SM [35]. In particular, the ZZh_p coupling becomes maximal in this limit. One can then detect the production of h_p in the process

$$e^+e^- \longrightarrow Zh_p \tag{77}$$

If m_h falls well below the bound (75), the ZZh_p coupling might be very small, in which case the rate for reaction (77) becomes too small to be useful. However, one can then look for

$$e^+e^- \longrightarrow h_p A_p \qquad \text{or} \qquad e^+e^- \longrightarrow ZH_p. \tag{78}$$

By searching for reactions (77) and (78) together, one can cover the entire parameter space of the MSSM [36], *provided* one has an e^+e^- collider with center–of–mass energy $\sqrt{s} \geq 300$ GeV. No such collider exists as yet; however, there are plans in various countries to build a linear e^+e^- collider

[11] Note that the relevant top mass in eq.(75) is the running $\overline{\text{MS}}$ mass taken at scale $\sqrt{m_t \cdot m_{\tilde{t}}}$ [33], which is some 10 GeV smaller than the pole mass $m_t(\text{pole}) = 175 \pm 6$ GeV [34].

[12] The alert reader might wonder why the form of the correction (75) is so different from the corrections we computed in Sec. 2, eq.(9). The reason is that eq.(75) has been derived by requiring that the vevs remain fixed. The corrections $\propto m_{\tilde{t}}^2$ to the Higgs mass parameter m_2^2 that we found in Sec. 2 are therefore absorbed by a change of the tree–level value of that parameter. Of course, at some point this will lead to unacceptable finetuning; this is why one usually does not consider stop masses (greatly) exceeding 1 TeV. After this procedure, the dominant corrections to the *physical* Higgs masses only grow logarithmically with the stop masses, as indicated in eq.(75).

with $\sqrt{s} \geq 500$ GeV [37]. If experiments at such a collider fail to find at least one Higgs boson, the MSSM can be completely excluded, independent of the values of its 100 or so free parameters.

In fact, an only slightly weaker version of this statement holds in *all* models with weak–scale supersymmetry, *if* one requires that all couplings of the theory remain in the perturbative regime, i.e. *if* the theory remains weakly coupled, up to some very high energy scale of order of the GUT scale $M_X \simeq 10^{16}$ GeV. In such more general models, which introduce new Higgs self couplings by introducing Higgs singlets, the upper bound (76) could increase to something like 150 GeV [38]; it remains true, however, that an e^+e^- collider with $\sqrt{s} \geq 300$ GeV has to discover at least one Higgs boson [39].

The last physical Higgs boson of the MSSM is the charged $H_p^\pm$, with mass

$$m^2_{H^\pm} = M^2_W + m^2_A; \tag{79}$$

notice that it is always heavier than the W boson.[13] The case $m_A^2 \gg M_Z^2$ is of particular interest. In this "decoupling limit" A_p, H_p and $H_p^\pm$ are all very close in mass; they essentially form a degenerate $SU(2)$ doublet. Furthermore, m_h is close to its upper bound and, as mentioned earlier, the couplings of h_p approach those of the single Higgs boson of the SM. Since in this scenario h_p would be the only Higgs boson that can be discovered at the next round of colliders [35], it would be difficult to distinguish between the SM and the MSSM by just studying the Higgs sector. However, one could still deduce that the SM should cease to describe Nature at a relatively low energy scale, beyond which "new physics" of some sort has to appear. The reason is that in the SM the renormalization group running of the quartic Higgs coupling would lead to the scalar potential becoming unbounded from below at a scale $\Lambda \ll M_X$, if the Higgs mass is below 150 GeV or so [40]. Searches for Higgs bosons therefore play a very important role in testing SUSY in general and the MSSM in particular. So far, the most stringent bounds on the Higgs sector come from experiments at the e^+e^- collider LEP [23]:

$$m_h \geq 62 \text{ GeV}, \qquad \text{if } m_A^2 \gg M_Z^2; \tag{80a}$$

$$m_h,\ m_A \geq 45 \text{ GeV}, \qquad \text{if } m_A \simeq m_h. \tag{80b}$$

This concludes my discussion of $SU(2) \times U(1)_Y$ breaking in the MSSM. More detailed studies can be found e.g. in refs.[35].

4c. Sparticle Mixing

Once $SU(2) \times U(1)_Y$ is broken, fields with different $SU(2) \times U(1)_Y$ quantum numbers can mix, if they have the same $SU(3)_c \times U(1)_{\rm em}$ quantum numbers. The Dirac masses of the SM quarks and leptons can be understood as such mixing terms, since they couple a left–handed $SU(2)$ doublet to a right–handed singlet. A closely related phenomenon occurs in the sfermion sector of the MSSM.

All three types of contributions to the scalar potential (F–terms, D–terms and SUSY breaking terms) appear in the sfermion mass matrices. I will for the moment ignore mixing between sfermions of different generations, but will include mixing between $SU(2)$ doublet and singlet sfermions. The sfermion mass matrix then decomposes into a series of 2×2 matrices, each of which describes sfermions of a given flavor. Let us consider the case of the scalar top. The F–term contribution

[13] This is not necessarily true in more general SUSY models [38].

(39) gives rise to diagonal $\tilde{t}_L$ and $\tilde{t}_R$ masses from $\left|\partial f/\partial \tilde{t}_R\right|^2$ and $\left|\partial f/\partial \tilde{t}_L\right|^2$, respectively; these contributions are equal to m_t^2. F–terms also give an off–diagonal contribution from $\left|\partial f/\partial \bar{H}^0\right|^2$, which is proportional to $\lambda_t v \mu = m_t \mu \cot\beta$. The D–terms (44) only give rise to diagonal mass terms, but these differ for $\tilde{t}_L$ and $\tilde{t}_R$, since these fields transform differently under $SU(2) \times U(1)_Y$. Finally, the soft breaking terms (61) give (in general different) contributions to the diagonal $\tilde{t}_L$ and $\tilde{t}_R$ mass terms, as well as an off–diagonal contribution $\propto A_t m_t$. Altogether one has [41] [in the basis ($\tilde{t}_L$, $\tilde{t}_R$)]:

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} m_t^2 + m^2_{\tilde{t}_L} + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)\cos(2\beta) M_Z^2 & -m_t\left(A_t + \mu\cot\beta\right) \\ -m_t\left(A_t + \mu\cot\beta\right) & m_t^2 + m^2_{\tilde{t}_R} + \frac{2}{3}\sin^2\theta_W \cos(2\beta) M_Z^2 \end{pmatrix}. \tag{81}$$

Note that the off–diagonal entries are $\propto m_t$. Eq.(81) also describes the $\tilde{c}$ and $\tilde{u}$ mass matrices, with the obvious replacement $t \rightarrow c$ or u. Since m_c, $m_u \ll m_{\tilde{c}}$, $m_{\tilde{u}}$, $\tilde{u}_L - \tilde{u}_R$ and $\tilde{c}_L - \tilde{c}_R$ mixing are usually negligible.[14] However, since the top mass is comparable to the other masses that appear in eq.(81), $\tilde{t}_L$–$\tilde{t}_R$ mixing is generally important. To mention but one example, even though the $Z\tilde{t}_L\tilde{t}_L^*$ and $Z\tilde{t}_R\tilde{t}_R^*$ couplings are nonzero, the coupling of the Z boson to a physical stop eigenstate of the matrix (81), $\tilde{t}_1 = \tilde{t}_L \cos\theta_{\tilde{t}} + \tilde{t}_R \sin\theta_{\tilde{t}}$, vanishes [42] if $\cos^2\theta_{\tilde{t}} = \frac{4}{3}\sin^2\theta_W$.

The calculation of the sbottom mass matrix is completely analogous to that of $\mathcal{M}^2_{\tilde{t}}$. Ohe has [in the basis ($\tilde{b}_L$, $\tilde{b}_R$)]:

$$\mathcal{M}^2_{\tilde{d}} = \begin{pmatrix} m_b^2 + m^2_{\tilde{t}_L} - \left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right)\cos(2\beta) M_Z^2 & -m_b\left(A_b + \mu\tan\beta\right) \\ -m_b\left(A_b + \mu\tan\beta\right) & m_b^2 + m^2_{\tilde{b}_R} - \frac{1}{3}\sin^2\theta_W \cos(2\beta) M_Z^2 \end{pmatrix}. \tag{82}$$

Note that the soft breaking mass that appears in the $(1,1)$ entry of $\mathcal{M}^2_{\tilde{b}}$ is the same as that in the $(1,1)$ entry of $\mathcal{M}^2_{\tilde{t}}$. This is a consequence of $SU(2)$ invariance: If the SUSY breaking piece (61) of the Lagrangian contained different masses for members of the same doublet, $SU(2)$ would be broken explicitly and the theory would no longer be unitary on the quantum level. This leads to the important relation

$$m^2_{\tilde{l}_L} = m^2_{\tilde{\nu}_l} - M_W^2 \cos(2\beta), \tag{83}$$

which holds if $\tilde{l}_L - \tilde{l}_R$ mixing can be neglected; this is always the case for $l = e$ or μ. However, in general no such relation holds for masses of $SU(2)$ singlet sfermions.

The off–diagonal entries of the matrix (82) are again proportional to the relevant quark mass. Nevertheless, $\tilde{b}_L - \tilde{b}_R$ (as well as $\tilde{\tau}_L - \tilde{\tau}_R$) mixing can be important [43] if $\tan\beta \gg 1$. Such scenarios are viable, since the b quark is the heaviest fermion that gets its mass from the vev v, which therefore need not be larger than a few GeV. Note that the top mass is $\propto \bar{v}$. This implies that the top and bottom Yukawa couplings will be close to each other if $\tan\beta \simeq m_t/m_b \simeq 50$; this is necessary in certain Grand Unified models based on the group $SO(10)$ [44]. Notice, however, that $\tilde{b}_L$–$\tilde{b}_R$ mixing, if it is important at all, is driven by μ, while the dominant contribution to $\tilde{t}_L$–$\tilde{t}_R$ mixing usually comes from A_t. ($A_{b,t} \gg m_{\tilde{b},\tilde{t}}$ is forbidden, since it leads to a charge and colour breaking absolute minimum of the scalar potential [45].)

Mixing between $\tilde{t}_L$ and $\tilde{t}_R$ and, if $\tan\beta \gg 1$, between $\tilde{b}_L$ and $\tilde{b}_R$ as well as between $\tilde{\tau}_L$ and $\tilde{\tau}_R$ is quite generic. In contrast, mixing between sfermions of different generations is very model

[14] An exception can occur for (loop) processes where chirality arguments imply that *all* contributions are suppressed by small quark masses; in such cases the off–diagonal entries in the $\tilde{u}$ and $\tilde{c}$ mass matrices must be included.

dependent. Such mixing can cause severe phenomenological problems, by producing unacceptably large flavor changing neutral currents (FCNC) between ordinary quarks and leptons through 1–loop processes. Such problems occur if the mass matrix for squarks with a given charge does not commute with the corresponding quark mass matrix, since then there will be flavor off–diagonal gluino–quark–squark couplings. This can easily be seen by writing the physical (mass) eigenstates as $(q_p)_i = \sum_j (U_q)_{ij} q_j$ and $(\tilde{q}_p)_i = \sum_j (U_{\tilde{q}})_{ij} \tilde{q}_j$, where q_j and $\tilde{q}_j$ are current eigenstates. This gives [see eq.(40)]:

$$\begin{aligned} \mathcal{L}_{\tilde{g}\tilde{q}q} &\propto \overline{\tilde{g}} \sum_{i=1}^{3} q_i \tilde{q}_i^* + h.c \\ &= \overline{\tilde{g}} \sum_{i=1}^{3} \left[\sum_{j=1}^{3} \left(U_q^\dagger \right)_{ij} (q_p)_j \right] \cdot \left[\sum_{l=1}^{3} \left(U_{\tilde{q}}^T \right)_{il} \left(\tilde{q}_p^* \right)_l \right] + h.c. \\ &= \overline{\tilde{g}} \sum_{j,l=1}^{3} \left(U_{\tilde{q}} U_q^\dagger \right)_{lj} (q_p)_j \left(\tilde{q}_p^* \right)_l + h.c. \end{aligned} \tag{84}$$

This will be flavor–diagonal only if the matrices U_q and $U_{\tilde{q}}$ can be chosen to be equal, which is possible if the q and $\tilde{q}$ mass matrices commute. This condition is trivially satisfied if squarks of a given charge all have the same mass, in which case their mass matrix is proportional to the unit matrix, but other possibilities also exist. A recent analysis [46] finds that constraints on $K^0 - \overline{K^0}$ and $D^0 - \overline{D^0}$ mixing force the off–diagonal entries in eq.(84) between the first and second generation to be very small, unless squarks and gluinos are significantly heavier than 1 TeV. The corresponding bounds involving third generation (s)quarks are somewhat weaker.

Similar problems also arise in the slepton sector, if one replaces the gluino in eq.(84) with a bino or neutral wino. In this case the most severe constraints come from $\mu \to e\gamma$ decays and $\mu \to e$ conversion in muonic atoms. Finally, chargino loop contributions to $K^0 - \overline{K^0}$ mixing limit mass splitting between $SU(2)$ doublet squarks of the first and second generation, (almost) independently of any mixing angles [47].

The breaking of $SU(2) \times U(1)_Y$ also leads to mixing between electroweak gauginos and higgsinos. This mixing is caused by the last term in eq.(40), which can couple a Higgs boson to a gaugino and a higgsino; when the Higgs field is replaced by its vev, these terms give rise to off–diagonal entries in the "chargino" and "neutralino" mass matrices. The physical charginos $\tilde{\chi}_{1,2}^+$ are therefore mixtures of the charged $SU(2)$ gauginos and the charged higgsinos. Their mass matrix in the (gaugino, higgsino) basis can be written as

$$\mathcal{M}_\pm = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}. \tag{85}$$

Notice that $\mathcal{M}_\pm$ is not symmetric, unless $\tan\beta = 1$. In general one therefore needs two different diagonalization matrices for the right– and left–handed components of the charginos; see the first paper in ref.[35] for a careful discussion of this point.

The neutralinos are mixtures of the $\widetilde{B}$, the neutral $\widetilde{W}$, and the two neutral higgsinos. In general these states form four distinct Majorana fermions, which are eigenstates of the symmetric mass matrix [in the basis $(\tilde{B},\ \widetilde{W},\ \tilde{h}^0,\ \tilde{\bar{h}}^0)$]:

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin\theta_W & M_Z\sin\beta\sin\theta_W \\ 0 & M_2 & M_Z\cos\beta\cos\theta_W & -M_Z\sin\beta\cos\theta_W \\ -M_Z\cos\beta\sin\theta_W & M_Z\cos\beta\cos\theta_W & 0 & -\mu \\ M_Z\sin\beta\sin\theta_W & -M_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix}. \tag{86}$$

The masses and mixing angles of the charginos and neutralinos are therefore completely determined by the values of the four parameters $M_1,\ M_2,\ \mu$ and $\tan\beta$. In most analyses one further reduces the dimensionality of this parameter space by assuming that gaugino masses unify at the GUT scale $M_X \simeq 10^{16}$ GeV. This is motivated by the observation that in the MSSM the three gauge couplings do seem to meet at this scale, if one uses their experimentally determined values as inputs at scale M_Z and "runs" them to higher scales using their renormalization group equations (RGE) [48]. The gaugino masses M_i run in the same way as the corresponding squared gauge couplings g_i^2 do. Assuming $M_1 = M_2$ at scale M_X then implies

$$M_1(M_Z) = \frac{5}{3}\tan^2\theta_W M_2(M_Z) \simeq \frac{1}{2}M_2(M_Z), \tag{87}$$

where the factor 5/3 comes from the difference between the GUT normalization and the usual SM normalization of the hypercharge generator.

If eq.(87) holds, the phenomenology of the charginos and neutralinos is essentially fixed by three parameters.[15] If $|\mu| > |M_2| \geq M_Z$, the two lightest neutralino states will be dominated by the gaugino components, with $\tilde{\chi}_1^0$ being mostly $\tilde{B}$ and $\tilde{\chi}_2^0$ being mostly $\widetilde{W}^0$; similarly, the light chargino $\tilde{\chi}_1^\pm$ will be mostly a charged wino. In this case one very roughly finds $m_{\tilde{\chi}_1^\pm} \simeq m_{\tilde{\chi}_2^0} \simeq 2m_{\tilde{\chi}_1^0}$. In the opposite limit, $|\mu| < |M_1|$, the two lighter neutralinos and the lighter chargino are all mostly higgsinos, with masses close to $|\mu|$. Finally, if $|\mu| \simeq |M_2|$, some of the states will be strongly mixed. The size of the mixing also depends to some extend on $\tan\beta$. If $\tan\beta$ is not large, there will be considerably more mixing if $M_2\cdot\mu\cdot\tan\beta > 0$ than for the opposite choice of sign; notice that for this sign, the two terms in the determinant of the chargino mass matrix tend to cancel, and a similar cancellation occurs in $\det\mathcal{M}_0$. Such mixing lowers the mass of the light eigenstates, and increases that of the heavy ones.

The details of the chargino and neutralino sectors are of importance for many areas of MSSM phenomenology. For example, $\tilde{\chi}_1^0$ is usually taken to be the LSP; we saw in Sec. 4a that the LSP must be electrically and color neutral. This means that any other sparticle will eventually decay into a $\tilde{\chi}_1^0$. However, in many cases the $\tilde{\chi}_1^0$ is produced only at the end of a lenghty decay chain or "cascade" [49]. Consider the case of the gluino. If gaugino masses are unified, the gluino mass $|M_3|$ at the weak scale is about 3.5 times larger than $|M_2|$. The gluino will therefore decay into the lighter electroweak gauginos and a pair of quarks via the exchange of real or virtual squarks. However, since $g_2^2 \simeq 3.3g_1^2$, gluinos will prefer to decay into the $SU(2)$ gaugino states, even though these are heavier than the $U(1)_Y$ gaugino. If $|\mu| < |M_2|$, gluinos will therefore decay into the *heaviest* chargino and neutralinos, which in turn will decay by the exchange of gauge or Higgs boson or a sfermion; often these decays will themselves proceed in several steps. The situation can become even more complicated if top quarks can be produced in gluino decays, since the (s)top has large

[15] Their decay branching ratios in general also depend on sfermion and Higgs masses, however.

Yukawa couplings to higgsino–like charginos and neutralinos. A realistic treatment of gluino decays has to keep track of all these different possibilities. Space does not permit to delve into these details any further here; I refer the interested reader to ref.[25], where many more references can be found.

4d. Minimal Supergravity

As mentioned at the end of Sec. 4a, a general parametrization of supersymmetry breaking in the MSSM introduces about 100 free parameters. This is not very satisfactory. The predictive power of a theory with such a large number of parameters is clearly quite limited, although we saw in Sec. 4b that some interesting predictions can be derived even in this general framework. It is therefore desirable to try and construct models that make do with fewer free parameters. The oldest and, to my mind, still most elegant such model goes under the name minimal supergravity (mSUGRA).

This model is based on the *local* version of supersymmetry. Eqs.(12b) show that invariance under local SUSY transformations implies invariance under local coordinate change; this invariance is the principle underlying Einstein's construction of the theory of General Relativity. Local supersymmetry therefore naturally includes gravity, and is usually called supergravity.

We saw in Sec. 2f that it is quite difficult to break global supersymmetry spontaneously, partly because this necessarily creates a cosmological constant. This is no longer true for supergravity. To see that, one first introduces the "Kähler potential"

$$G = -\sum_i X_i(\phi_j)\phi_i\phi_i^* - M_{Pl}^2 \log \frac{|f(\phi_j)|^2}{M_{Pl}^6}, \tag{88}$$

where f is again the superpotential, and the X_i are real functions of the chiral fields ϕ_j. Supersymmetry is broken spontaneously if $\langle G_i \rangle \equiv \langle \partial G/\partial \phi_i \rangle \neq 0$ for some i; in the "flat limit" $M_{Pl} \to \infty$ this reduces to $\langle \partial f/\partial \phi_i \rangle \neq 0$, which is the condition for F–term breaking of global supersymmetry, see eq.(56). For finite M_{Pl}, the F–term contribution to the scalar potential becomes

$$V_F = M_{Pl}^2 e^{-G/M_{Pl}^2} \left[G^i \left(G^{-1} \right)_i^j G_j + 3 M_{Pl}^2 \right], \tag{89}$$

where $G^i \equiv \partial G/\partial \phi_i^*$, and $(G^{-1})_i^j$ is the matrix of second derivatives of G. There can be a cancellation between the two terms in the square bracket in eq.(89); one can therefore simultaneously have $\langle G_i \rangle \neq 0$ (broken SUSY) and $\langle V_F \rangle = 0$ (vanishing cosmological constant).

It is nevertheless still very difficult to break supersymmetry using only the fields present in the MSSM. One therefore introduces a "hidden sector" [50], which consists of some fields that do not have any gauge or superpotential couplings to the "visible sector" containing the MSSM. Nevertheless, the supergravity Lagrangian [7] *automatically* transmits SUSY breaking from the hidden to the visible sector, through operators that are suppressed by some powers of M_{Pl}. In the simplest models, the order of magnitude of the soft breaking terms (61) will be set by the gravitino mass,

$$m_{3/2} = M_{Pl} e^{-G/(2M_{Pl}^2)} = \frac{|\langle f \rangle|}{M_{Pl}^2} \exp\left(\sum_i X_i \phi_i \phi_i^* / M_{Pl}^2 \right). \tag{90}$$

The most natural choice is to give some hidden sector field(s) vev(s) of order M_{Pl}. Then $\langle f \rangle \sim \mathcal{O}(M_{Pl} \langle f_i \rangle)$, and the second factor in eq.(90) is of order unity. Requiring the soft breaking masses to be $\sim \mathcal{O}(M_Z)$ then implies

$$\langle f_i \rangle \sim \mathcal{O}(M_Z \cdot M_{Pl}) \sim \left(10^{10}\ \mathrm{GeV}\right)^2. \tag{91}$$

However, one can also construct models where the soft breaking masses in the visible sector are either much smaller or much larger than the gravitino mass [51].[16]

The ansatz (88) is not very predictive; in general it still gives different soft terms for different scalar fields [52, 11]. The number of free parameters can, however, be reduced dramatically by imposing a global $U(N)$ symmetry on the "Kähler metric" $(G^{-1})^i_j$, where N is the number of superfields in the visible sector (17 in the MSSM, for three generations plus two Higgs doublets). This implies in particular that the functions X_i must be the same for all MSSM fields. One then finds [53] that SUSY breaking in the visible sector can be described using only three parameters:

$$m_i^2 = m_0^2 \qquad \forall i \tag{92a}$$

$$A_{ijk} = A_0 \qquad \forall i,j,k \tag{92b}$$

$$B_{ij} = B_0 \qquad \forall i,j \tag{92c}$$

In the MSSM there is in any case only one $B-$parameter; however, eqs.(92a,b) are very restrictive indeed. Finally, in mSUGRA one assumes that the gaugino masses are unified at the GUT scale:

$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}. \tag{93}$$

In principle eqs.(92) should hold at an energy scale close to M_{Pl}, where gravitational interactions are integrated out. In practice one usually assumes that these "boundary conditions" still hold at the GUT scale M_X. This is not a bad approximation if there is no full GUT field theory, e.g. if the MSSM directly merges into superstring theory (which also predicts unification of gauge couplings). However, if a GUT exists, eqs.(92) will in general receive sizable corrections [54]. Unfortunately these corrections depend quite sensitively on the poorly known details of the GUT sector.

Even if eqs.(92) still hold to good approximation at scale M_X, the spectrum at the weak scale is significantly more complicated. The reason is that the soft breaking parameters "run", i.e. depend on the energy scale, just like the gauge couplings do. Indeed, we already saw in the previous subsection that the gaugino masses have the same scale dependence as the squared gauge couplings; eq.(93) therefore implies

$$M_a(Q) = m_{1/2} \frac{g_a^2(Q)}{g_a^2(M_X)}, \tag{94}$$

so that $M_3 : M_2 : M_1 \simeq 7 : 2 : 1$ at the weak scale.

Scalar masses also receive corrections from gauge interactions, involving both gauge boson loops and gaugino–fermion (or, in case of Higgs bosons, gaugino–higgsino) loops, as discussed in Sec. 2e. In fact, each class of contributions by itself is quadratically divergent, but these divergencies cancel between the bosonic and fermionic contributions. However, if SUSY is broken, a logarithmic

[16] Formally [7], the "SUSY breaking scale" $M_S^2 = -M_{Pl} e^{-\langle G \rangle/(2M_{Pl}^2)} \left\langle (G^{-1})^i_j G_i \right\rangle$. Note that $(G^{-1})^i_j = -\delta^i_j$ for fields with canonical kinetic energy terms. This gives $M_S^2 = e^{-X\phi^2/M_{Pl}^2} \left[\frac{\langle X \phi_i f \rangle}{M_{Pl}^2} + \langle f_i \rangle \right]$. Under the "natural" assumptions listed above, this reproduces approximately the numerical value given in eq.(91).

divergence remains, which contributes to the running of the scalar masses:[17]

$$\left.\frac{dm_i^2}{d\log Q}\right|_{\rm gauge} = -\sum_{a=1}^{3} \frac{g_a^2}{8\pi^2} c_a(i) M_a^2, \tag{95}$$

where a labels the gauge group, and $c_a(i)$ is a group factor; e.g. $c_3(\tilde{q}) = 16/3$ for $SU(3)$ triplets, $c_2(\tilde{l}_L) = 3$ for $SU(2)$ doublets, and $c_1(\phi) = 4Y_\phi^2$ for fields with hypercharge Y_ϕ. Note the negative sign in eq.(95); it implies that gauge interactions will *increase* scalar masses as the scale Q is reduced from M_X to the weak scale. Obviously the sfermions with the strongest gauge interactions will receive the largest corrections. This leads to the predictions

$$1 \leq m_{\tilde{q}}/m_{\tilde{l}_L} \leq 3.5; \tag{96a}$$

$$1 \leq m_{\tilde{l}_L}/m_{\tilde{l}_R} \leq 1.9, \tag{96b}$$

where the lower bounds hold for $m_{1/2} \rightarrow 0$ and the upper bounds for $m_0 \rightarrow 0$. Note that eqs.(96) hold only if Yukawa interactions are negligible, which is true for the first two generations, as well as for the stau's and $\tilde{b}_R$ unless $\tan\beta \gg 1$.

The effect of Yukawa interactions on the running of Higgs masses has almost been computed in Sec. 2, eq.(11); the only missing piece is a contribution from the wave function renormalization of the Higgs field, which is proportional to the Higgs mass itself. Altogether, one finds for the $\bar{H}$ doublet [55]:

$$\frac{dm_{\bar{H}}^2}{d\log Q} = \frac{3\lambda_t^2}{8\pi^2}\left(m_{\bar{H}}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2\right) + (\text{gauge terms}). \tag{97}$$

Since the second Higgs doublet, H, only has Yukawa interactions involving λ_b and λ_τ, radiative corrections from Yukawa interactions give *different* contributions to m_H^2 and $m_{\bar{H}}^2$. This is important, since we saw in Sec. 4b, eqs.(64) and (65), that successful $SU(2) \times U(1)_Y$ breaking requires $m_H^2 \neq m_{\bar{H}}^2$ at the weak scale, whereas the boundary condition (92a) stipulates their equality at the Planck or GUT scale. Note also that the sign in eq.(97) is positive. This means that Yukawa interactions will *reduce* $m_{\bar{H}}^2$ as we go down from M_X to M_Z. In fact, given that $\lambda_t \geq 1$ at the weak scale, $m_{\bar{H}}^2$ can quite easily become negative at scales Q exponentially smaller than M_X, thereby triggering $SU(2) \times U(1)_Y$ breaking [55, 56]. This is called "radiative symmetry breaking", since radiative corrections as described by eq.(97) play a crucial role here.[18]

The stop mass parameters $m_{\tilde{t}_R}^2$ and $m_{\tilde{t}_L}^2$ also receive corrections $\propto \lambda_t^2$. However, the group (color) factor 3 in eq.(97) is reduced to 2 and 1, respectively, since for stops the color index is fixed on the external legs; the correction to $m_{\tilde{t}_R}^2$ receives a factor of two due to summation over $SU(2)$ indices in the loop. The group structure of the MSSM therefore naturally singles out the Higgs fields, since their masses are reduced most by radiative corrections involving Yukawa interactions. Notice also that $m_{\tilde{t}_{L,R}}^2$ in general receive large positive corrections from $SU(3)$ gauge interactions, while the gauge terms in eq.(97) only involve the much weaker $SU(2) \times U(1)_Y$ interactions.

[17] Note that the logarithmic divergencies we found in Sec. 3e should be absorbed in the running of the $SU(2)$ gauge coupling, not the running of the Higgs mass.

[18] One occasionally sees the statement in the literature that $\lambda_t \geq 1$, i.e. a heavy top quark, is necessary for this mechanism to work. This is not true, since condition (65) can be satisfied even if both m_H^2 and $m_{\bar{H}}^2$ are positive. Indeed, for historical reasons there is an extensive literature [57] on radiative symmetry breaking with a top mass around 40 GeV. It is true, however, that the allowed parameter space opens up as λ_t is increased.

Eq.(97) implies that the Higgs mass parameters at the weak scale depend in a quite complicated fashion on the GUT–scale parameters m_0, $m_{1/2}$, A_0 and B_0, as well as on the top Yukawa coupling. Of course, we still want to arrange things such the Z boson gets its proper mass. One way to ensure this is to chose as free parameters the set (m_0, $m_{1/2}$, A_0, $\tan\beta$, signμ), and to use eqs.(69) to determine B and $|\mu|$ at the weak scale. This is a convenient choice since (at least at one–loop level) the RGE for B and μ are decoupled from those for the other soft breaking parameters [55]. The value of B is of little interest per se, since this parameter only appears in the Higgs potential. The parameter μ also appears in many mass matrices, however; see eqs.(81), (82), (85) and (86). Since λ_t is large, $m_{\tilde{H}}^2$ usually becomes quite negative at the weak scale, and one needs a sizable μ^2 in eq.(69b); hence one usually (but not always) has $|\mu| \geq |M_2|$ at the weak scale [43, 58]. An exception can occur for $m_0^2 \gg m_{1/2}^2$, where $|\mu| < m_{1/2}$ is still possible. Similarly, one finds that three of the four Higgs bosons are usually quite heavy. In particular, at tree–level the mass of the pseudoscalar boson is given by [43]

$$m_A^2 = \frac{m_{\tilde{\nu}}^2 + \mu^2}{\sin^2\beta} + \mathcal{O}(\lambda_b^2, \lambda_\tau^2). \tag{98}$$

One is thus usually close to the "decoupling limit" $m_A^2 \gg M_Z^2$ discussed in Sec. 4b. However, the Yukawa terms omitted in eq.(98) are negative; if $\tan\beta \simeq m_t/m_b$, they can reduce m_A to values at (or below) its experimental lower bound [43]. In fact, the requirement $m_A^2 > 0$ often determines the upper bound on $\tan\beta$.

Space does not allow me to extend this rather sketchy introduction to mSUGRA phenomenology. The interested reader is referred to refs.[25, 28] for further details (and many additional references). Note also that a program that implements radiative symmetry breaking starting from the boundary conditions (92), (93) is part of the ISAJET event generator [59]; copies of the program are available from `baer@hep.fsu.edu`.

5. Outlook

Clearly a short introduction to supersymmetry, like the one I have attempted here, can at best give a flavor of the work that has been, and is being, done in this very large field. In the main part of these lectures I had to give short thrift to most recent developments. In this concluding section I will try to at least briefly sketch some areas of active research, and provide some of the relevant references.

Recent efforts in SUSY model building, i.e. the construction of potentially realistic supersymmetric field theories, have mostly focussed on two quite different approaches. On the one hand, there has been much interest in models where supersymmetry is broken in a hidden sector at a rather low scale, and is then mediated to the visible sector by gauge interactions [17]. One drawback of such models is that one needs to introduce a new "messenger sector" to transmit SUSY breaking to the MSSM fields; in mSUGRA this is done *automatically* by terms in the Lagrangian whose presence is required by local supersymmetry. Proponents of this class of models list as its main advantage that it automatically gives equal masses to sfermion with equal $SU(3) \times SU(2) \times U(1)_Y$ quantum numbers, thereby avoiding the problems with FCNC discussed in Sec. 4b. In fact, however, this is not true if one writes down the most general superpotential allowed by the gauge symmetry of the MSSM; one has to introduce additional symmetries to forbid Yukawa couplings between the messenger and MSSM sectors.

Interest in this class of models peaked a few months ago, since they seemed to be able to explain a rather strange event [60] observed by the CDF collaboration at the tevatron $p\bar{p}$ collider, where the final state consists of an e^+e^- pair and two hard photons; the transverse momenta of these four particles do not add up to zero, i.e. there is missing transverse energy. The probability for this event to be due to SM processes is estimated to be of the order of 10^{-3}. Since in these models the SUSY breaking scale is rather low, the gravitino $\widetilde{G}$ (which does not have any gauge interactions) is very light, see eq.(90). In this case $\tilde{\chi}_1^0 \rightarrow \widetilde{G} + \gamma$ decays can have quite short lifetimes [61], and the "CDF event" could be explained as selectron pair production followed by $\tilde{e} \rightarrow \tilde{\chi}_1^0 + e$ and $\tilde{\chi}_1^0 \rightarrow \widetilde{G} + \gamma$ decays. However, this explanation is beginning to look quite unlikely, since one then expects (many) more unusual events in other final states involving hadronic jets [62], none of which seem to have been seen [63]. Also, it was recently pointed out that at least the simplest models in this class have absolute minima of the scalar potential where either SUSY remains unbroken in the MSSM sector, or charge and color are broken [64]. My personal view is that these models introduce needless complications with no apparent gain.[19]

In a quite different development, people have been trying to construct models that describe both the large mass splittings in the SM quark and lepton sector and the required near–degeneracy of their scalar superpartners (more accurately, the fact that the sfermion mass matrices have to almost commute with the fermion mass matrices) by the *same* mechanism. Usually this is achieved by means of a judiciously chosen discrete [65] or continuous [66] symmetry. Most of these models have been embedded in some GUT theory, so whatever new (s)particles they predict (beyond those of the MSSM) tend to have masses $\mathcal{O}(M_X)$. However, they also predict (at least "generically") that sfermion loop contributions to FCNC processes, while suppressed, are not vanishingly small. This gives new importance to searches for processes that are forbidden in the SM (e.g., $\mu \rightarrow e\gamma$ decays), as well as to careful experimental studies of processes that are allowed but suppressed in the SM [46] ($b \rightarrow s\gamma$ decays, $K^0 - \overline{K^0}$, $D^0 - \overline{D^0}$ and $B^0 - \overline{B^0}$ mixing, various CP–violating asymmetries, ...).

Finally, "string phenomenologists" attempt to make contact between superstring theory and the real world [67]. This approach might well be the most promising one in the long run. However, to the best of my knowledge no firm prediction of phenomenological interest has yet emerged from string theory.

The recent proliferation of SUSY models makes it imperative to devise ways to distinguish between them. The most direct method is to measure the masses and couplings of superparticles as accurately as possible. This has been the focus of many recent studies of SUSY collider phenomenology [68, 69]. Searching for superparticles at e^+e^- colliders is usually quite straightforward, since the relevant SM background processes usually have roughly comparable cross–sections; the presence of massive invisible LSPs in SUSY events then gives them kinematic properties that allow to distinguish them from SM backgrounds. However, when one is trying to find ways to measure MSSM parameters precisely, even small backgrounds can become important. Furthermore, one may have to distinguish experimentally between different SUSY processes, which can be much more challenging than discriminating between SUSY and SM events.

Hunting for sparticles at hadron colliders is more tricky, since now the cross section for (hard)

[19] I am a strong believer in Occam's razor, even though it may seem sexist nowadays.

SM processes often exceeds those for the SUSY reactions of interest by several orders of magnitude. Another problem is the relatively more "messy" environment in which experiments at hadron colliders must work. This is partly due to the fact that the initial state now radiates gluons (rather than photons at e^+e- colliders), each of which will produce several hadrons. Another problem is that only some fraction of the p or $\bar{p}$ beam energy goes into the hard (partonic) scattering reaction, with the rest going into "beam remnants". This makes kinematic event reconstruction much more difficult, since much of the beam remnants escapes down the beam pipes, carrying an unknown amount of energy and longitudinal momentum with it. Therefore one often only uses momentum components transverse to the beam in partial kinematic reconstructions, and also for the purpose of devising cuts that increase the signal–to–background ratio.

Nevertheless several promising signals for sparticle production at hadron colliders have been clearly established in Monte Carlo studies [25]. Unfortunately searches for these final states at existing colliders so far have yielded null results; one can conclude from these studies that (most) squarks and gluinos have to be heavier than about 200 GeV, the precise bounds depending on details like the squark to gluino mass ratio [14]. These limits are still well below the naturalness or finetuning limit of very roughly 1 TeV. However, at the LHC collider, which is scheduled to commence operations in about a decade, one should see clear SUSY signals in several different final states if supersymmetry is to provide a solution to the hierarchy problem. Work on how to measure MSSM parameters and distinguish between competing SUSY models at hadron collider experiments has only begun relatively recently [69].

Finally, a considerable amount of work has gone into studies of implications of supersymmetry for cosmology and astrophysics. The perhaps most prominent example is "Dark Matter" (DM) [70], which is known to form most of the mass of the Universe. Studies of Big Bang nucleosynthesis indicate that at least some of the DM must not be baryonic. The LSP, being absolutely stable if R–parity is unbroken, has been known for some time to make a good DM particle candidate [71]. This is now a relatively mature field [70], but calculations of the density of LSP relics from the Big Bang era, and studies of how to detect them experimentally, are still being refined. The first pilot searches [72] for DM particles already exclude the possibility that a stable sneutrino could be the LSP. These experiments were not sensitive enough to probe much of the MSSM parameter space if the LSP is the lightest neutralino, but this is expected to change in the next decade or so. Finally, without going into any detail I mention that superparticles might also play a crucial role in generating the baryon asymetry of the Universe [73, 74], and perhaps even in the conjectured very early "inflationary" stage of the development of the Universe [74].

It should be clear by now that supersymmetrists have penetrated almost all areas of particle physics research. You are welcome to join us!

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